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Fuzzy approximating functions and its application in solving fuzzy multi-choice linear programming models

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Abstract. This article considers a particular type of fuzzy multi-choice linear programming (FMCLP) model in which there are several choices for the fuzzy parameters on the right-hand side (RHS) of problem constraints. We first construct the fuzzy polynomials to solve this model using the fuzzy multi-choice parameters on the RHS of constraints. We construct the fuzzy polynomials by approximating fuzzy functions, including the binary variable approach, Lagrange, and Newton's interpolating polynomials. Also, we use the least squares approach to construct the approximating fuzzy polynomial. Then we solve the resulting model. Finally, we will examine the above techniques in numerical examples.

Keywords: Fuzzy multi-choice linear programming model, fuzzy binary variable approach, fuzzy interpolation polynomials, fuzzy least squares method.
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1 Introduction

Linear programming (LP) problem is a system process of finding a maximum or minimum value of a function which needs to be optimized subject to a set of different constraints. It is helpful in developing and solving a decision making problem by mathematical techniques.

An LP problem in deterministic state is defined as follows [32]

$$\begin{array}{lll} \min/\max & \sum_{j=1}^{n} c_j t_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} t_j \leq b_i, & i = 1,2,\ldots,m, \\ & t_j \geq 0, & j = 1,2,\ldots,n. \end{array}$$

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The parameters we use in the LP problem may be multi-choice, meaning that there are several choices for the desired parameter. This LP problem is called the multi-choice LP (MCLP) problem [10, 11, 17, 18, 28, 31].

The multi-choice programming (MCP) problems were first introduced, belonging to the class of combinational programming problems in which only one option is selected from a set of parameters to optimize the objective function. In [18, 28], the authors have devised a mathematical model in which they use binary variables to solve the problem of MCLP. Chang [11, 26] presented a solution to the problems of multi-choice goal programming (MCGP) problems. In another article by Chang [12], multi-choice constraints are replaced by a continuous function for various multi-choice parameters. In [10], the interpolating polynomials were used to solve the MCLP; the authors converted the MCLP into a nonlinear programming problem by replacing the multi-choice parameters with interpolating polynomials.

Fuzzy MCLP (FMCLP) is an MCLP in which some parameters, or all of them also decision variables, can be fuzzy numbers. Fuzzy MCGP, fuzzy multi-objective programming problem, multi-choice random transportation problem, multi-choice transportation problem with multi-choice cost and demand and random supply, multi-choice multi-objective transportation problem, and fuzzy MCLP in a fuzzy random hybrid uncertainty case have been studied by researchers [2, 4, 6, 11, 13, 15, 19, 21, 23–25, 29, 30, 35].

Here, we investigate the fuzzy MCLP in which the parameters on the RHS of constraints are fuzzy numbers. Without loss of generality, we investigate the triangular fuzzy numbers. In [27], FMCLP with some fuzzy parameters has been solved. At first, the fuzzy numbers are converted to crisp numbers using the ranking functions. Then the crisp MCLP, using the Lagrange polynomial, is converted to a nonlinear programming model. We intend to use fuzzy approximating functions in solving FMCLP. An important class of approximating functions is polynomials [5]; as polynomials are computationally simple. There are many types of fuzzy approximating polynomials. In our work, we use the approximating polynomials based on a binary variable approach [9], an interpolation-based including NDD, NBD, and NFD methods [3, 16]. Also, we apply the least-squares method. Our method converts the FMCLP to a classical fuzzy programming problem by approximating the fuzzy multi-choice parameters on the RHS of constraints with a fuzzy approximate function. Then we use the mathematical software to solve the obtained fuzzy linear programming problrm [35].

The remainder of the article includes the following subjects. In Section 2, the form of FMCLP that we consider, and its solution methods are proposed. In Section 3, we present the contents of solving FMCLP using fuzzy linear least squares method. In Section 4, examples, including two FMCLP, are solved using the proposed method. Finally, the conclusion is given.

2 FMCLP and its solution methods

In problems caused by real phenomena, we often encounter fuzzy data and problems. Fuzzy programming problems occur in many practical cases, [7], and they have different types. If all the problem's variables and their parameters are fuzzy, it is called a fully fuzzy problem [14], which is not discussed here. The FMCLP is a fuzzy programming problem where its parameters may be multi-choice. In this paper, we examine FMCLP, in which only the multi-choice parameters on the RHS of the problem constraints are fuzzy numbers.

We consider FMCLP as follows

$$
\begin{aligned}
\max \quad & y=\sum_{j=1}^{n} c_j t_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq \left\{\widetilde{b}_i^{(0)}, \widetilde{b}_i^{(1)}, \ldots, \widetilde{b}_i^{(n_i-1)}\right\}, \quad && i=1,\ldots,m, \\
& t_j \geq 0, && j=1,2,\ldots,n,
\end{aligned} \tag{1}
$$

where $\widetilde{b}_i^{(k)}=\left(l_i^{(k)}, m_i^{(k)}, r_i^{(k)}\right)$, $i=1,2,\ldots,m$, $k=0,1,\ldots,n_i-1$ are triangular fuzzy numbers. To solve problem (1), we make a fuzzy approximating polynomial, $\widetilde{p}$, based on the multi-choice data associated with the i-th constraint in model (1). In this way, FMCLP is transformed into a fuzzy LP problem as follows

$$
\begin{aligned}
\max \quad & y=\sum_{j=1}^{n} c_j t_j, \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq \widetilde{p}^{(i)}(z), \quad && i=1,\ldots,m, \\
& t_j \geq 0, && j=1,2,\ldots,n.
\end{aligned} \tag{2}
$$

To make $\widetilde{p}^{(i)}(z)$, we use different type of the approximating methods that are proposed in the next subsections.

2.1 Solving FMCLP using binary method

Using the mathematical model provided by Chang in which binary polynomials are defined according to the number of multi-choice parameters [8, 9]; for fuzzy mode we also need the same polynomials. The only difference between this case and the crisp case is that the polynomial's coefficients are fuzzy triangles numbers so the resulting binary polynomial is a fuzzy polynomial. In this method, the RHS of constraints in (1) is replaced with a continuous, binary fuzzy function. The RHS of the i-th constraint has n_i number of known fuzzy parameters where only one of them should be selected. The model is transformed into an equivalent model that can be solved by standard mathematical programming tools. We investigate the cases of $n_i=2^{m_i}$ and $n_i \neq 2^{m_i}$, where $n_i=1,2,3,4$. We will check some of the modes of this issue.

Case 1: $n_i=2$.
In this case, we have two known parameters, $\widetilde{b}_i^{(0)}$ and $\widetilde{b}_i^{(1)}$, where we select one of them. Since the total number of parameters is 2, we need only one binary variable $z^{(1)}$. By using $z^{(1)}$, the following model is obtained [9]

$$
\begin{aligned}
\max \quad & y=\sum_{j=1}^{n} c_j t_j, \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq z^{(1)} \widetilde{b}_i^{(0)}+\left(1-z^{(1)}\right) \widetilde{b}_i^{(1)}, \quad && i=1,2,\ldots,m, \\
& t_j \geq 0, && j=1,2,\ldots,n, \\
& z^{(1)}=0,1.
\end{aligned}
$$

Case 2: $n_i = 3$.
By selecting $n_i = 3$, we have three known parameters, $\widetilde{b}_i^{(0)}, \widetilde{b}_i^{(1)}$ and $\widetilde{b}_i^{(2)}$, where one of them is selected. Since the number of elements of the set is three and $2^1 < 3 < 2^2$, only two binary variables $z^{(1)}$, and $z^{(2)}$, are needed. We consider three as $\binom{2}{2} + \binom{2}{1}$ or $\binom{2}{1} + \binom{2}{0}$. Then we introduce an additional constraint in the model so we can obtain two following different models [9]

Model 1:

$$
\begin{aligned}
\max \quad & y = \sum_{j=1}^{n} c_j t_j, \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq \left(1 - z^{(1)}\right)\left(1 - z^{(2)}\right)\widetilde{b}_i^{(0)} + \left(1 - z^{(1)}\right) z^{(2)} \widetilde{b}_i^{(1)} + z^{(1)}\left(1 - z^{(2)}\right)\widetilde{b}_i^{(2)}, \quad && i = 1, 2, \ldots, m, \\
& t_j \geq 0, && j = 1, 2, \ldots, n, \\
& z^{(1)} + z^{(2)} \leq 1, \\
& z^{(1)}, z^{(2)} = 0, 1.
\end{aligned}
$$

Model 2:

$$
\begin{aligned}
\max \quad & y = \sum_{j=1}^{n} c_j t_j, \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq \left(1 - z^{(1)}\right) z^{(2)} \widetilde{b}_i^{(0)} + \left(1 - z^{(2)}\right) z^{(1)} \widetilde{b}_i^{(1)} + z^{(1)} z^{(2)} \widetilde{b}_i^{(2)}, \quad && i = 1, 2, \ldots, m, \\
& t_j \geq 0, && j = 1, 2, \ldots, n, \\
& z^{(1)} + z^{(2)} \geq 1, \\
& z^{(1)}, z^{(2)} = 0, 1.
\end{aligned}
$$

Case 3: $n_i = 2^2$.
In this case, one of the four known parameters, $\widetilde{b}_i^{(k)}$, $k = 0, 1, 2, 3$, is considered. The total number of choices is $\mathbf{2^2}$, so two binary variables, $z^{(1)}$ and $z^{(2)}$are needed. Using them, we can obtain the following model [9]:

$$
\begin{aligned}
\max \quad & y = \sum_{j=1}^{m} c_j t_j, \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} t_j \leq z^{(1)} z^{(2)} \widetilde{b}_i^{(0)} + \left(1 - z^{(1)}\right) z^{(2)} b_i^{(1)} + \left(1 - z^{(2)}\right) z^{(1)} \widetilde{b}_i^{(2)} \\
& \qquad + \left(1 + z^{(1)}\right)\left(1 - z^{(2)}\right)\widetilde{b}_i^{(3)}, && i = 1, 2, \ldots, m, \\
& t_j \geq 0, && j = 1, 2, \ldots, m \\
& z^{(1)}, z^{(2)} = 0, 1.
\end{aligned}
$$

The procedure for $n_i = 5, 6, 7, \ldots, 16$ is similar to the above cases. More details on these cases for deterministic case have been represented in [8, 9].

2.2 Solving FMCLP using fuzzy interpolation method

For solving FMCLP (1) by interpolation approach, we consider n_i disjoint numbers $0, 1, \ldots, n_i - 1$ as node points where $\widetilde{b}_i^{(0)}, \widetilde{b}_i^{(1)}, \ldots, \widetilde{b}_i^{(n_i-1)}$, are the associate fuzzy values of the data points. So, we have the following points

$$\left\{\left(0,\widetilde{u}_0=\widetilde{b}_i^{(0)}\right),\left(1,\widetilde{u}_1=\widetilde{b}_i^{(1)}\right),\ldots,\left(n_i-1,\widetilde{u}_{n_i-1}=\widetilde{b}_i^{(n_i-1)}\right)\right\}, \tag{3}$$

where, $\widetilde{b}_i^{(k)}=\left(l_i^{(k)},m_i^{(k)},r_i^{(k)}\right), i=1,2,\ldots,m,\ k=0,1,\ldots,n_i-1$. We make the interpolating polynomial $\widetilde{p}^{(i)}(z)=(l_i(z),m_i(z),r_i(z))$, $i=1,2,\ldots,m$, such that $\widetilde{p}^{(i)}(k)=\widetilde{b}_i^{(k)}$, $i=1,2,\ldots,m$, $k=0,1,\ldots,n_i-1$. In this way, problem (1) converts to

$$\begin{array}{lll}\max & y=\sum\limits_{j=1}^{n} c_j t_j, & \\ \text{s.t.} & \sum\limits_{j=1}^{n} a_{ij} t_j \leq \widetilde{p}^{(i)}(z), & i=1,2,\ldots,m, \\ & z=0,1,\ldots,n_i-1, & i=1,2,\ldots,m, \\ & t_j \geq 0, & j=1,2,\ldots,n.\end{array}$$

The fuzzy interpolating polynomial $\widetilde{p}^{(i)}(z)$ is formulated by several methods below.

2.2.1 Fuzzy Lagrange polynomial

The fuzzy Lagrange polynomial for the support points in (3) is defined as follow [20, 22]:

$$\begin{aligned}\tilde{p}^{(i)}(z) &= (l_i(z),m_i(z),r_i(z))=\sum_{k=0}^{n_i-1} L_k(z)\,\tilde{u}_k=\sum_{k=0}^{n_i-1} L_k(z)\,(l_i^{(k)},m_i^{(k)},r_i^{(k)}) \\ &= \sum_{L_k(z)\geq 0}(L_k(z)l_i^{(k)},L_k(z)\,m_i^{(k)},L_k(z)\,r_i^{(k)})+\sum_{L_k(z)<0}(L_k(z)r_i^{(k)},L_k(z)\,m_i^{(k)},L_k(z)\,l_i^{(k)}),\end{aligned}$$

where $L_k(z)=\prod\limits_{\substack{k=0\\k\neq i}}^{n_i-1}\dfrac{z-k}{i-k}$, $i=1,\ldots,m$ are basic functions, and

$$\begin{aligned}l_i(z) &= \sum_{L_k(z)\geq 0} L_i(z)l_i^{(k)}+\sum_{L_k(z)<0} L_k(z)r_i^{(k)}, \\ m_i(z) &= \sum_{k=0}^{n_i-1} L_k(z)\,m_i^{(k)}, \\ r_i(z) &= \sum_{L_k(z)\geq 0} L_k(z)r_i^{(k)}+\sum_{L_k(z)<0} L_k(z)l_i^{(k)}.\end{aligned}$$

2.2.2 Fuzzy Newton's interpolating polynomial

Consider $(k,\tilde{u}_k)$, $k=0,\ldots,n_i-1$, where $\tilde{u}_k$ is a fuzzy number. If we want to get the fuzzy Newton's interpolating polynomials for this data, we need the difference between two fuzzy numbers as follows.

Definition 1. *[34] Let $\widetilde{a}=(a_1,a_2,a_3)$ and $\widetilde{b}=(b_1,b_2,b_3)$ be two triangular fuzzy numbers. Then the difference between them is defined as follows $\widetilde{a}-\widetilde{b}=(a_1-b_3,a_2-b_2,a_3-b_1)$.*

The problem is that the difference between two fuzzy numbers changes the interpolation data, to solve this obstacle, we use the Hukuhara difference.

Definition 2. *(Hukuhara difference [33]). Suppose* $\widetilde{a} = (a_1, a_2, a_3)$*,* $\widetilde{b} = (b_1, b_2, b_3)$ *are triangular fuzzy numbers. The Hukuhara difference* $\widetilde{b}, \widetilde{a}$*,* $\widetilde{b} \ominus_H \tilde{a}$ *, is defined as* $\widetilde{b} \ominus_H \widetilde{a} = \widetilde{c} \Rightarrow \widetilde{b} = \widetilde{a} \oplus \widetilde{c}$*, where* $\widetilde{c} = (c_1, c_2, c_3)$ *is triangular fuzzy number and we have*

$$(b_1, b_2, b_3) = (a_1 + c_1, a_2 + c_2, a_3 + c_3) \Rightarrow \widetilde{c} = (b_1 - a_1, b_2 - a_2, b_3 - a_3).$$

However, the Hukuhara difference between two fuzzy numbers will not necessarily be a fuzzy number. For example, consider $\widetilde{a} = (15, 19, 22)$ *,* $\widetilde{b} = (17, 18, 20)$*, then* $\widetilde{b} \ominus_H \widetilde{a} = (2, 1, -2)$ *, is not a fuzzy number. We use the generalized Hukuhara difference to solve this issue.*

Definition 3. *(Generalized Hukuhara difference). The generalized Hukuhara difference between* $\widetilde{b} = (b_1, b_2, b_3)$*,* $\widetilde{a} = (a_1, a_2, a_3)$ *is denoted by* $\widetilde{b} \ominus_{GH} \widetilde{a}$ *and defined as follows [33]:*

$$\widetilde{b} \ominus_{GH} \widetilde{a} = \widetilde{c} \Rightarrow \begin{cases} \widetilde{b} = \widetilde{a} \oplus \tilde{c}, \\ or \\ \widetilde{a} = \widetilde{b} \oplus (-\widetilde{c}). \end{cases}$$

In other words,

$$(b_1, b_2, b_3) - (a_1, a_2, a_3) = (c_1, c_2, c_3) \Rightarrow \begin{cases} \widetilde{c} = (b_1 - a_1, b_2 - a_2, b_3 - a_3), \\ or \\ (b_3 - a_3, b_2 - a_2, b_1 - a_1), \end{cases}$$
$$\Rightarrow \widetilde{c} = (\min\{b_1 - a_1, b_3 - a_3\}, b_2 - a_2, \max\{b_3 - a_3, b_1 - a_1\}).$$

In some cases, using the generalized Hukuhara difference does not solve the above mentioned issue. For example, consider two fuzzy numbers $\widetilde{a} = (15, 19, 20)$ *,* $\widetilde{b} = (17, 18, 20)$*, then*

$$\widetilde{c} = \widetilde{b} \ominus_{GH} \widetilde{a} = (\min\{17 - 15, 20 - 20\}, 18 - 19, \max\{17 - 15, 20 - 20\}) = (2, -1, 2).$$

It is evident $\widetilde{c}$ *is not a fuzzy number. In these cases, the generalized fuzzy number is used.*

Definition 4. *(Generalized fuzzy number). The fuzzy number* $\widetilde{a} = (m, \alpha, \beta)_{LR}$ *is called a generalized fuzzy number (GLR-fuzzy number) if it has one of the following forms [14]*

I. if $\alpha < 0, \beta > 0 \Rightarrow \widetilde{a} = (m, 0, \max\{-\alpha, \beta\})_{GLR}$*,*
II. if $\alpha > 0, \beta < 0 \Rightarrow \widetilde{a} = (m, \max\{\alpha, -\beta\}, 0)_{GLR}$*,*
III. if $\alpha < 0, \beta < 0 \Rightarrow \widetilde{a} = (m, -\beta, -\alpha)_{GLR}$*,*
IV. if $\alpha > 0, \beta > 0 \Rightarrow \widetilde{a} = (m, \alpha, \beta)_{GLR}$*.*

Using Definition 3, for two fuzzy numbers $\widetilde{a} = (15, 19, 20)$*,* $\widetilde{b} = (17, 18, 20)$*, where* $\widetilde{c} = \tilde{b} \ominus_{GH} \tilde{a} = (2, -1, 2)$ *is not a fuzzy number, using Definition 4 we can write* $\widetilde{c} = (2, -1, 2) = (-1 - (-3), -1, -1 + 3)$*, i.e.*

$$\alpha = -3, \beta = 3 \Rightarrow \widetilde{A} = (m, 0, \max\{-\alpha, \beta\})_{GLR} = (-1, 0, 3)_{GLR} = (-1, -1, 2)_{GLR}.$$

Therefore, based on the above, we use the generalized Hukuhara difference to obtain the difference of two fuzzy numbers. If the obtained number is not a fuzzy number, we use the generalized fuzzy numbers definition.

Now, we can obtain fuzzy Newton's interpolating polynomial. The fuzzy NDD interpolating polynomial is defined as follows

$$\widetilde{p}(z) = \widetilde{u}_0 + z\widetilde{u}[0,1] + z(z-1)\widetilde{u}[0,1,2] + \cdots + z(z-1)\ldots(z-n_i+2)\widetilde{u}[0,1,,\ldots,n_i-1],$$

where

$$\begin{aligned}
\widetilde{u}[0,1] &= \widetilde{u}_1 - \widetilde{u}_0 = \widetilde{b}_i^{(1)} - \widetilde{b}_i^{(0)},\\
\widetilde{u}[0,1,2] &= \frac{\widetilde{u}[1,2] - \widetilde{u}[0,1]}{2},\\
&\vdots\\
\widetilde{u}[0,1,\ldots,j] &= \frac{\tilde{u}[1,2,\ldots,j] - \widetilde{u}[0,1,\ldots,j-1]}{j}.
\end{aligned}$$

Because the nodes in (3) are arranged with equal spacing $h = 1$, the fuzzy NFD interpolating polynomial is formulated as follows:

$$\widetilde{p}(z) = \widetilde{u}_0 + z\Delta\widetilde{u}_0 + \frac{(z)(z-1)}{2!}\Delta^2\widetilde{u}_0 + \cdots + \frac{(z)(z-1)\ldots(z-n_i+2)}{(n_i-1)!}\Delta^{n_i-1}\tilde{u}_0,$$

where for $k = 0,1,\ldots,n_i-2$, we have

$$\begin{aligned}
\Delta\widetilde{u}_k &= \widetilde{u}_{k+1} - \widetilde{u}_k,\\
\Delta^2\tilde{u}_k &= \Delta\tilde{u}_{k+1} - \Delta\tilde{u}_k,\\
&\vdots\\
\Delta^p\widetilde{u}_k &= \Delta^{p-1}\widetilde{u}_{k+1} - \Delta^{p-1}\tilde{u}_k.
\end{aligned}$$

Because the nodes in (3) are arranged with equal spacing $h = 1$, the fuzzy NBD interpolating polynomial is formulated as follows:

$$\widetilde{p}(z) = \widetilde{u}_{n_i-1} + z\nabla\widetilde{u}_{n_i-1} + \frac{(z)(z+1)}{2!}\nabla^2\widetilde{u}_{n_i-1} + \cdots + \frac{(z)(z+1)\ldots(z+n_i-2)}{(n_i-1)!}\nabla^{n_i}\widetilde{u}_{n_i-1},$$

where for $k = 1,\ldots,n_i-1$, we have

$$\begin{aligned}
\nabla\widetilde{u}_k &= \widetilde{u}_k - \widetilde{u}_{k-1},\\
\nabla^2\widetilde{u}_k &= \nabla\widetilde{u}_k - \nabla\widetilde{u}_{k-1},\\
&\vdots\\
\nabla^p\widetilde{u}_k &= \nabla^{p-1}\widetilde{u}_k - \nabla^{p-1}\widetilde{u}_{k-1}.
\end{aligned}$$

Now, in problem (1), we approximate the RHS of the problem constraints using fuzzy Newton's interpolating polynomial. In this way, problem (1) is converted to (2). This problem is a fuzzy LP in the RHS we have a fuzzy polynomial with integer variables. We solve this problem according to the degree of necessity that the decision maker determines.

3 Solving FMCLP using fuzzy linear least squares method

Consider the fuzzy function $\tilde{u}(z)$ in the points $0,1,\ldots,n_i-1$. The goal is to find the fuzzy linear least squares method so that the sum of the squares of the vertical distance $(k,\tilde{u}_k)$, $k=0,\ldots,n_i-1$ of the points of line $\tilde{p}(z)=\tilde{a}_0+\tilde{a}_1 z$ is the minimum value, where $\tilde{a}_0$, and $\tilde{a}_1$ are the triangular fuzzy numbers are as follows: $\widetilde{a}_0=(a_0^1,a_0^2,a_0^3)$, $\widetilde{a}_1=(a_1^1,a_1^2,a_1^3), \widetilde{u}_k=(u_k^1,u_k^2,u_k^3)$ and $\widetilde{p}$ is a triangular fuzzy linear polynomial.

If we denote the fuzzy least-squares error by $\widetilde{\mathrm{E}}$, then

$$\widetilde{E}=\sum_{k=0}^{n_i-1}(\widetilde{u}_k-\widetilde{a}_0-\widetilde{a}_1 z_k)^2=\sum_{k=0}^{n_i-1}(\widetilde{u}_k-\widetilde{a}_0-\widetilde{a}_1 k)^2.$$

For being $\widetilde{E}$ minimum, it is necessary and sufficient that its fuzzy derivatives respect to $\widetilde{a}_0$ and $\widetilde{a}_1$ is equal to zero, in which case the following conditions are obtained:

$$\begin{cases} \widetilde{a}_0(\sum\limits_{k=0}^{n_i-1}k)+\widetilde{a}_1(\sum\limits_{k=0}^{n_i-1}k^2)=\sum\limits_{k=0}^{n_i-1}k\widetilde{u}_k, \\ \widetilde{a}_0\,(n_i)+\widetilde{a}_1(\sum\limits_{k=0}^{n_i-1}k)=\sum\limits_{k=0}^{n_i-1}\widetilde{u}_k. \end{cases}$$

In other words

$$\begin{cases} (a_0^1,a_0^2,a_0^3)\,(\sum\limits_{k=0}^{n_i-1}k)+(a_1^1,a_1^2,a_1^3)(\sum\limits_{k=0}^{n_i-1}k^2)=\sum\limits_{k=0}^{n_i-1}k(u_k^1,u_k^2,u_k^3), \\ (a_0^1,a_0^2,a_0^3)\,(n_i)+(a_1^1,a_1^2,a_1^3)\left(\sum\limits_{k=0}^{n_i-1}k\right)=\sum\limits_{k=0}^{n_i-1}\left(u_k^1,u_k^2,u_k^3\right). \end{cases} \tag{4}$$

By comparing the two sides of (4), the following system is obtained:

$$\begin{cases} a_0^1\left(\sum\limits_{k=0}^{n_i-1}k\right)+a_1^1\left(\sum\limits_{k=0}^{n_i-1}k^2\right)=\sum\limits_{k=0}^{n_i-1}ku_k^1, \\ a_0^2\left(\sum\limits_{k=0}^{n_i-1}k\right)+a_1^2\left(\sum\limits_{k=0}^{n_i-1}k^2\right)=\sum\limits_{k=0}^{n_i-1}ku_k^2, \\ a_0^3\left(\sum\limits_{k=0}^{n_i-1}k\right)+a_1^3\left(\sum\limits_{k=0}^{n_i-1}k^2\right)=\sum\limits_{k=0}^{n_i-1}ku_k^3, \\ a_0^1 n_i+a_1^1\left(\sum\limits_{k=0}^{n_i-1}k\right)=\sum\limits_{k=0}^{n_i-1}u_k^1, \\ a_0^2 n_i+a_1^2\left(\sum\limits_{k=0}^{n_i-1}k\right)=\sum\limits_{k=0}^{n_i-1}u_k^2, a_0^3 n_i+a_1^3\left(\sum\limits_{k=0}^{n_i-1}k\right)=\sum\limits_{k=0}^{n_i-1}u_k^3. \end{cases} \tag{5}$$

The unknown values a_0^1, a_0^2, a_0^3, a_1^1, a_1^2, and a_1^3 will be obtained by solving system (5). In this case, we get the line $\widetilde{p}$ as $\widetilde{p}(z) = (l(z), m(z), r(z)) = (a_0^1 + a_1^1 z, a_0^2 + a_1^2 z, a_0^3 + a_1^3 z)$.

4 Numerical examples

In this section, two FMCLPs are considered to investigate the efficiency of the presented approach. As we said earlier, we consider FMCLPs in which there are several choices for the fuzzy parameters on the RHS of problem constraints. We first construct the fuzzy polynomials to solve this model using the fuzzy multi-choice parameters on the RHS of constraints. Then we solve the resulting fuzzy model by Lingo with the degree of necessity of 0.8. The polynomials we used are constructed based on the binary variable approach, Lagrange, NDD, NBD, and NFD interpolating polynomials and linear least squares approach.

Example 1. Consider the following FMCLP

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{7} c_j t_j \\
\text{s.t.} \quad & t_1 + t_4 + t_5 + t_6 + t_7 \geq \{(13,14,16)\,(12,16,17)\,(15,17,20)\}, \\
& t_1 + t_2 + t_5 + t_6 + t_7 \geq \{(10,12,15), (12,13,14)\}, \\
& t_1 + t_2 + t_3 + t_6 + t_7 \geq \{(12,14,15), (13,15,18)\,(16,17,18), (17,19,21)\}, \\
& t_1 + t_2 + t_3 + t_4 + t_7 \geq \{(13,15,18), (14,16,18), (16,18,20), (17,19,21), (20,21,22)\}, \\
& t_1 + t_2 + t_3 + t_4 + t_5 \geq \{(12,14,15)\}, \\
& t_2 + t_3 + t_4 + t_5 + t_6 \geq \{(14,15,16), (15,17,20)\}, \\
& t_3 + t_4 + t_5 + t_6 + t_7 \geq \{(10,11,12), (12,14,15)\,(13,15,16)\}, \\
& t_j = 1,2,\ldots n, \qquad j = 1,2,\ldots,7.
\end{aligned}
$$

The mathematical model based on the binary polynomials is defined as follows

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{7} c_j t_j, \\
\text{s.t} \quad & t_1 + t_4 + t_5 + t_6 + t_7 \geq \big(13 - 14z^{(1)}z^{(2)} + 2z^{(1)} - z^{(2)}, 14 - 19z^{(1)}z^{(2)} + 3z^{(1)} \\
& \qquad\qquad + 2z^{(2)}, 16 - 21z^{(1)}z^{(2)} + 4z^{(1)} + z^{(2)}\big), \\
& t_1 + t_2 + t_5 + t_6 + t_7 \geq (12 - 2z^{(3)}, 13 - z^{(3)}, 14 + z^{(3)}), \\
& t_1 + t_2 + t_3 + t_4 + t_7 \geq \big(-4z^{(5)} - z^{(4)} + 17, z^{(4)}z^{(5)} - 4z^{(5)} - 3z^{(4)} + 19, -3z^{(5)} - 3z^{(4)} + 21\big), \\
& t_1 + t_2 + t_3 + t_4 + t_7 \geq (13z^{(6)} + 14z^{(7)} + 16z^{(8)} - 10z^{(6)}z^{(7)} - 29z^{(6)}z^{(8)} \\
& \qquad\qquad - 13z^{(7)}z^{(8)} + 6z^{(6)}z^{(7)}z^{(8)}, \\
& \qquad\qquad 15z^{(6)} + 16z^{(7)} + 18z^{(8)} - 12z^{(6)}z^{(7)} - 33z^{(6)}z^{(8)} \\
& \qquad\qquad - 13z^{(7)}z^{(8)} + 9z^{(6)}z^{(7)}z^{(8)}, \\
& \qquad\qquad 18z^{(6)} + 18z^{(7)} + 20z^{(8)} - 15z^{(6)}z^{(7)} - 38z^{(6)}z^{(8)} \\
& \qquad\qquad - 16z^{(7)}z^{(8)} + 9z^{(6)}z^{(7)}z^{(8)}), \\
& t_1 + t_2 + t_3 + t_4 + t_5 \geq (12,14,15), \\
& t_2 + t_3 + t_4 + t_5 + t_6 \geq \big(15 - z^{(9)}, 17 - 2z^{(9)}, 20 - 4z^{(9)}\big), \\
& t_3 + t_4 + t_5 + t_6 + t_7 \geq (10 + 3z^{(10)} + 2z^{(11)} - 15z^{(10)}z^{(11)}, 12 + 4z^{(10)} + 3z^{(11)} - 19z^{(10)}z^{(11)}), \\
& z^{(1)} + z^{(2)} \leq 1, \qquad z^{(6)} + z^{(7)} + z^{(8)} \geq 1, \qquad z^{(6)} + z^{(7)} + z^{(8)} \leq 2 \\
& z^{(6)} + z^{(8)} \leq 1, \qquad z^{(10)} + z^{(11)} \leq 1, z^{(1)},\ z^{(2)},\ z^{(3)}, z^{(4)}, z^{(5)}, z^{(6)}, z^{(7)}, z^{(8)}, z^{(9)}, z^{(10)}, z^{(11)} = 0,1.
\end{aligned}
$$

The mathematical model based on the Lagrange polynomials is defined as follows

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{7} c_j t_j, \\
\text{s.t.} \quad & t_1+t_4+t_5+t_6+t_7 \geq (13-3z_1+2z_1^2, 14+2.5z_1-0.5z_1^2, 16+z_1^2) \\
& t_1+t_2+t_5+t_6+t_7 \geq (10-z_2, 12+z_2, 15-z_2), \\
& t_1+t_2+t_3+t_6+t_7 \geq \big(12-4.3z_3+5.5z_3^2+1.7z_3^3, 14+0.17z_3+z_3^2 \\
& \qquad\qquad -0.17z_3^3, 15-6.5z_3-4.5z_3^2+z_3^3\big), \\
& t_1+t_2+t_3+t_4+t_7 \geq (13-2.17z_4+3.79z_4^2-1.5z_4^3+0.21z_4^4, 15 \\
& \qquad\qquad -1.17z_4+3.3z_4^2-2.3z_4^3+0.18z_4^4, 18-3z_4+5.8z_4^2 \\
& \qquad\qquad -1.5z_4^3-0.16z_4^4), \\
& t_1+t_2+t_3+t_4+t_5 \geq (12, 14, 15), \\
& t_2+t_3+t_4+t_5+t_6 \geq (14+z_6, 15+2z_6, 16+4z_6), \\
& t_3+t_4+t_5+t_6+t_7 \geq (10+2.5z_7-0.5z_7^2, 11+4z_7-z_7^2, 12+4z_7-z_7^2), \\
& z_1 = 0,1,2, z_2 = 0,1, z_3 = 0,1,2,3, z_4 = 0,1,2,3,4, z_6 = 0,1,\ z_7 = 0,1,2.
\end{aligned}
$$

By using fuzzy NDD interpolating polynomial, we obtain the following model

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{7} c_j t_j, \\
\text{s.t.} \quad & t_1+t_4+t_5+t_6+t_7 \geq (13-0.5z_1-0.5z_1^2, 14+2.5z_1-0.5z_1^2, 16+z_1+z_1^2), \\
& t_1+t_2+t_5+t_6+t_7 \geq (10+2z_2, 12+z_2, 15-z_2), \\
& t_1+t_2+t_3+t_6+t_7 \geq \big(12+1.16z_3-0.16z_3^3, 14+0.16z_3+z_3^2-0.16z_3^3, 15 \\
& \qquad\qquad +2.83z_3+0.16z_3^3-2.5z_3^3\big), \\
& t_1+t_2+t_3+t_4+t_7 \geq (13-2.5z_4+3.83z_4^2-1.5z_4^3+0.16z_4^4, 15-1.16z_4 \\
& \qquad\qquad +3.33z_4^2-1.33z_4^3+0.16z_4^4, 18-1.916z_4+4.291z_4^2 \\
& \qquad\qquad -1.583z_4^3+0.208z_4^4), \\
& t_1+t_2+t_3+t_4+t_5 \geq (12, 14, 15), \\
& t_2+t_3+t_4+t_5+t_6 \geq (14+z_6, 15+2z_6, 16+4z_6), \\
& t_3+t_4+t_5+t_6+t_7 \geq (10+3z_7-z_7^2, 11+4z_7-z_7^2, 12+3.5z_7-0.5z_7^2), \\
& z_1 = 0,1,2, \qquad z_2 = 0,1, \qquad z_3 = 0,1,2,3, \\
& z_4 = 0,1,2,3,4, \qquad z_6 = 0,1, \qquad z_7 = 0,1,2.
\end{aligned}
$$

Also, by using the fuzzy linear least squares method, we obtain the following model

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{7} c_j t_j \\
\text{s.t.} \quad & t_1+t_4+t_5+t_6+t_7 \geq (12.33+z_1, 14.16+1.5z_1, 17.33+z_1), \\
& t_1+t_2+t_5+t_6+t_7 \geq (10+2z_2, 12+z_2, 15-z_2), \\
& t_1+t_2+t_3+t_6+t_7 \geq (11.8+1.8z_3, 13.7+1.7z_3, 15.3+1.8z_3), \\
& t_1+t_2+t_3+t_4+t_7 \geq (12.6+1.7z_4, 14.8+1.5z_4, 17.6+1.1z_4), \\
& t_1+t_2+t_3+t_4+t_5 \geq (12, 14, 15) \\
& t_2+t_3+t_4+t_5+t_6 \geq (14+z_6, 15+2z_6, 16+4z_6), \\
& t_3+t_4+t_5+t_6+t_7 \geq (9.16+2.5z_7, 11.33+2z_7, 12.33+2z_7), \\
& z_1 = 0,1,2, \qquad z_2 = 0,1, \qquad z_3 = 0,1,2,3, \\
& z_4 = 0,1,2,3,4, \qquad z_6 = 0,1, \qquad z_7 = 0,1,2.
\end{aligned}
$$

Table 1 shows the solution of Example 1 by the binary method. Also, the results of interpolation and least squares methods are shown in Table 2.

Table 1: Solution of Example 1 by the binary method.

Method	y	t_1	t_2	t_3	t_4	t_5	t_6	t_7	$z^{(1)}$	$z^{(2)}$	$z^{(3)}$	$z^{(4)}$	$z^{(5)}$	$z^{(6)}$	$z^{(7)}$	$z^{(8)}$	$z^{(9)}$	$z^{(10)}$	$z^{(11)}$
Fuzzy binary polynomial	22	5	5	1	6	0	4	1	0	0	0	1	1	0	1	0	1	0	0

Table 2: Solution of Example 1 by the interpolation and least square schemes.

Method	y	t_1	t_2	t_3	t_4	t_5	t_6	t_7	z_1	z_2	z_3	z_4	z_6	z_7
Fuzzy Lagrange interpolating	21	2	7	0	6	0	3	3	2	1	0	0	0	0
Fuzzy Newton interpolating polynomial	22	3	4	2	6	0	4	3	0	1	0	0	0	0
Fuzzy linear least square method	22	4	2	0	7	0	4	2	0	1	0	0	0	0

Since the problem is a minimization problem, from Tables 1, and 2, we can conclude that the solution obtained in all methods are consistent.

Example 2. Consider the following FMCLP [1]

$$\begin{array}{ll} \max & y = 15t_1 + 16t_2 + 17t_3 + 12t_4, \\ \text{s.t.} & 10t_1 + 11t_2 + 12t_3 + 15t_4 \leq \{(270,271,280),(410,411,425),(570,573,578)\}, \\ & 14t_1 + 18t_2 + 17t_3 + 14t_4 \leq \{(380,385,390),(535,539,540)\}. \end{array} \tag{6}$$

To solve this model, we first make the fuzzy approximating polynomial based on the RHS of the constraints and then solve the resulting model.

The mathematical model based on the binary polynomials is defined as follows

$$\begin{array}{ll} \max & 15t_1 + 16t_2 + 17t_3 + 12t_4, \\ \text{s.t.} & 10t_1 + 11t_2 + 12t_3 + 15t_4 \leq (270 + 300z^{(1)}, +140z^{(2)} - 710z^{(1)}z^{(2)}, 271 + 302z^{(1)}, \\ & \qquad +140z^{(2)} - 713z^{(1)}z^{(2)}, 280 + 298z^{(1)}, +140z^{(2)} - 718z_1z_2) \\ & 14t_1 + 18t_2 + 17t_3 + 14t_4 \leq (535 - 155z^{(3)}, 539 - 154z^{(3)}, 540 + 150z^{(3)}), \\ & z_1 + z_2 \ \leq 1, \qquad z^{(1)}, z^{(2)}, z^{(3)} = 0,1. \end{array}$$

By using introduced fuzzy Lagrange interpolating polynomials, model (6) is converted to the following model

$$\begin{array}{ll} \max & 15t_1 + 16t_2 + 17t_3 + 12t_4, \\ \text{s.t.} & 10t_1 + 11t_2 + 12t_3 + 15t_4 \leq (10{z_1}^2 + 130z + 270, 11{z_1}^2 + 129z_1 + 271, 9{z_1}^2 + 131z_1 + 280), \\ & 14t_1 + 18t_2 + 17t_3 + 14t_4 \leq (155z_2 + 380, 154z_2 + 385, 150z_2 + 390), \\ & t_j \geq 0, \qquad j = 1,2,3,4, \\ & z_1 = 0,1,2, \qquad z_2 = 0,1. \end{array}$$

By using fuzzy NDD interpolating polynomial, we obtain the following model

$$\begin{array}{ll} \max & 15t_1+16t_2+17t_3+12t_4, \\ \text{s.t.} & 10t_1+11t_2+12t_3+15t_4 \leq (270+133.5z_1+6.5z_1^2, 271+129z_1+11z_1^2, 280+133z_1+11z_1^2), \\ & 14t_1+18t_2+17t_3+14t_4 \leq (155z_2+380, 154z_2+385, 150z_2+390), \\ & z_1=0,1,2, \qquad z_2=0,1. \end{array}$$

Also, by using fuzzy linear least squares method, we obtain the following model

$$\begin{array}{ll} \max & y=15t_1+16t_2+17t_3+12t_4, \\ \text{s.t.} & 10t_1+11t_2+12t_3+15t_4 \leq (150z_1+266.6, 151z_1+277.3, 149z_1+277), \\ & 14t_1+18t_2+17t_3+14t_4 \leq (155z_2+380, 154z_2+385, 150z_2+390), \\ & t_j \geq 0, \qquad j=1,2,3,4, \\ & z_1=0,1,2, \qquad z_2=0,1. \end{array}$$

Tables 3, and 4 show the solution of Example 2 by different methods.

Table 3: Solution of Example 2 by the binary method.

Method	y	t_1	t_2	t_3	t_4	$z^{(1)}$	$z^{(2)}$	$z^{(3)}$
Fuzzy binary polynomial	578.55	38.57	0	0	0	0	1	0

Table 4: Solution of Example 2 by the interpolation and least squares schemes.

Method	y	t_1	t_2	t_3	t_4	z_1	z_2
Fuzzy Lagrange interpolating polynomial	512.7	34.18	0	0	0	2	1
Fuzzy Newton interpolating polynomial	577.5	38.5	0	0	0	1	1
Fuzzy linear least-squares	578.35	38.55	0	0	0	1	1

Since the problem is a maximization problem, Tables 3, and 4 show that the objective values obtained from the least-squares and binary methods are almost the same and better than the other method.

5 Conclusions

In this paper, first we introduced the MCLP problems, in which there are several choices for the parameter on the RHS of the constraints. To solve these models, we replaced a multi-choice fuzzy parameter in the RHS of the constraints with a fuzzy polynomial. This polynomial is made by different approaches, including the binary variable approach, Lagrange, Newton interpolating polynomials, and the linear least-squares method. Then, we solved the resulting model with the degree of necessity 0.8 by Lingo software. By comparing the obtained solutions, we observed that the solution obtained in all methods are consistent. The Lagrange form of the fuzzy interpolating polynomial is not very convenient computationally. Instead, the Newton form of the interpolating polynomials is characterized by a cheaper computational cost. Also, using the fuzzy Newton interpolating polynomials increases the flexibility of the solution method, and if there is a change in the number of choices, for example some choices are added or deleted, the previous problem can be easily re-solved with a little change, and this is very effective in reducing computational costs. The fuzzy linear least squares method is easier to use than the fuzzy interpolation polynomials, but it ignores some choices, while in the interpolation, all the choices are taken into account. Using the fuzzy binary polynomial also takes into account all the choices, but using of these polynomials has many complexities and it is difficult to express and analyze the results using it.

This article examined the FMCLP problems in which only the RHS of the constraints is multi-choice fuzzy parameters. We intend to consider a case that all the problem variables and their coefficients are fuzzy numbers, and the RHS of the constraints are fuzzy multi-choice parameters for future research.

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