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A new mathematical model for reconstruction ECG signal based on non-polynomial cubic spline

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Abstract. Electrocardiogram (ECG) signals is widely used as one of the common procedures for heart's disease diagnose. Since electrical signals generated by biological sources have low level, they are destroyed by interference. Therefore, it is difficult to achieve high resolution electrical signals. A new approach based on non-polynomial cubic spline has been developed to approximate the ECG signal. The Efficiency of proposed method is analyzed by simulation results and filter evaluation metrics.

Keywords: ECG signal, noise, non-polynomial cubic spline, mathematical model.
AMS Subject Classification 2010: 92B05,41A15, 03C40.

1 Introduction

Cardiovascular diseases (CVDs) are common causes of death in the worldwide based on the world health organizations report (WHO) [1]. The heart conduction systems contain particular muscle cells. The general system is surrounded in myocardium. Main component of conduction systems consist of the SA node, AV node, bundle of His, bundle of branches and Purkinje fiber (see Fig. 1). Electrical impulses are created by the action of these component and spread all over the heart and cause it contract. Besides, any disorder of conduction systems effect in the regularity and the speed of heart rhythms which causes fast, slow or irregular. The electrical activity of heart can be measured by electrodes which are placed at particular position on the skin. The produced result is recorded in the form of a graph or ECG [13].

The electrical function of the heart has been modeled using reaction-diffusion equations. The influx of ions through the cell membrane (ionic currents) causes electric activation to propagate through nerve fibers, i.e. from ion movement inside or beyond the extracellular space. These ionic currents are illustrated as the reaction kinetics which is combined with the diffusion equation for the membrane potential. The Barrio-Varea-Aragon-Maini (BVAM) model is a generic reaction-diffusion system derived based on mass conservation of two morphogens and a Taylor expansion around an equilibrium point with cubic

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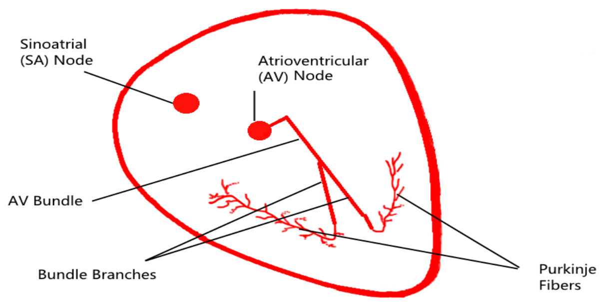

Figure 1: Heart electrical conduction system

non-linearity. This model has been applied to a broad range of trends seen in biological and chemical processes. In its simplest form, the BVAM model is as follows:

$$\begin{aligned} \frac{\partial u}{\partial t} &= D\nabla^2 u + \eta \left(u + av - Cuv - uv^2\right), \\ \frac{\partial v}{\partial t} &= \nabla^2 v + \eta \left(bv + Hu + Cuv + uv^2\right), \end{aligned} \tag{1}$$

where $u(x,t)$ and $v(x,t)$ denote two interacting variables with a constant diffusion coefficient ratio D at position x and time t. Depending on the method to be modeled, these variables may be interpreted as chemical elements, morphogens, or some other observable amount [16].

Since electrical signals generated by biological sources have low level, they are destroyed by interference. They are polluted by different kind of noises. Therefore, Noise omission from ECG signal is an impressive step for recovery resolution of signal in order to accurate visual interpretation. The application of mathematical models in medicine is growing rapidly [3, 10, 20].ECG signal is not separate from this fact. Using analog or digital filters for removing these artifacts is prevalent method. Due to the lack of instrumentation in digital filters, they are more precise compared with analog ones [12]. Some procedures mainly consist of infinite impulse response (IIR) [9], finite impulse response (FIR) [5,25], adaptive filter [7,11,26]. Although these methods can remove noise outside the ECG signals, they are incapable if the range of noise overlaps with the ECG signal. Besides, the Gibbs phenomenon is likely to occur when the order of the filter does not choose correctly. Other methods mainly include discrete wavelet transform [2,22,23], Principal Component Analysis (PCA) [8], Independent Component Analysis (ICA) [15], and Empirical Mode Decomposition (EMD) [4, 6] in order to ECG denoising. Baseline wander noise is a low frequency resulted in breathing, changing electrode impedance due to body movement. numerous methods have been proposed for reduction Baseline wander noise. Common methods applied for reduction Baseline wander noise are based on Fourier decomposision [24] and wavelet transformation [17,21]. Every method has special basis function [14]. Polynomial functions are more popular in compared with other ones because there are flexibility in applying nonlinear function. Besides, interpolation method for denoising ECG signal based on maximum degree 3, second degree polynomials, Legendre polynomials with high degree compression were applied as the first methods of approximation ECG signal [27].

Non-polynomial cubic spline polynomials are used in a variety of mathematical modeling and nu-

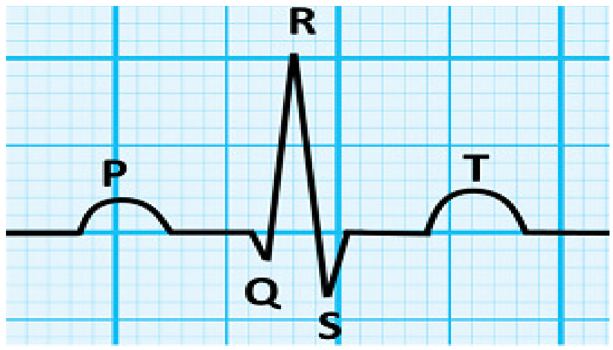

Figure 2: Simple model of ECG signal.

merical solution of ordinary differential equation and Partial differential equation but they are seldom used to improve signal quality by minimizing artifacts. The noise in the MIT-BIH database ECG signals is minimized in this paper using the Non-polynomial cubic Spline interpolation technique. According to our knowledge , using nonpolynomial cubic spline as an interpolant has been used for the first time so far. The numerical results illustrate capability of proposed method in this field.

In this paper, a new mathematical model is proposed in order to modification of ECG signal resolution based on interpolation technique. This procedure consists of two steps i.e. Moving Average Filter(MAF) and Non-polynomial Cubic Spline (NCS), respectively.

Advantages and disadvantages of procedure are discussed in detail. Therefore, this technique can be used for similar situation in the field of signal processing.

This paper is organized as follows. Section 2, is dedicated to introduced ECG signal. Section 3, Moving Average Filter, Non-polynomial Cubic Spline method is introduced. Section 4, evaluation metrics are introduced. Section 5, suggested methods are approximated real ECG data and the results are compared with each other geometrically and algebraically. Section 6, superiority of the proposed method is concluded.

2 ECG wave

The ECG displays hearts the electrical activity and applied as one of the most important information sources to physicians in order to diagnosis of heart diseases. Fig. 2 displays a simple model of ECG signal which is divided to three main segmentation P, QRS and T. This wave is produced by the contraction and expansion of the heart muscle. There is a relationship between type of heart function and each part of this category. P wave, QRS complex and T wave in one ECG complete wave are related to depolarization of the atria, excitation of ventricles and ventricular re-polarization, respectively. ECG signal samples which are used in our experiments are given from MIT-BIH Database and consists of baseline wander noise (0.18Hz) and power line interference (50Hz). MIT-BIH is a popular data base for this target.

3 Proposed method

High resolution ECG signals plays decisive rols in order to accurate diagnosis of the disease. Proposed algorithm in order to removal noise is implemented as follows: first of all, Moving Average Filter secondly (MAF) are enforced over noisy ECG signals in order to reduce noises. Secondly, the results of implementation of MAF are reconstructed by non-polynomial cubic spline (NCS) interpolation. In the other words,ECG signals are reconstructed by NCS. Generally, this procedures display by MNCS notation. Presented method is capable for reducing low level frequency noises such as baseline wander noise.

3.1 Moving average filter

Compression of signal is counted as an effective procedure for pre-processing. The benefits of using pre-processing is presentation striking features similar to inflexion and extrema points. One of the most prevalent process for data compression is the MOVING AVERAGE FILTER (MAF) which used to demonstrate significant trends in repeat statistical data. The main benefit of AVERAGE FILTER is diminishing random noise while sustaining striking features. Assume $S=\{x_1,x_2,\ldots,x_m,\ldots\}$ is the value of ECG sample, following algorithm illustrates how this filter works.

Algorithm 1. *Moving Average Filter.*
Input *:* $X=\{x_1,x_2,\ldots,x_n\}$ *.*
Step 1*: Set* $n=length(X)$ *and* $N\in\mathbf{N}, N<n$ *such that* N *is fix.*
Step 2*: Set* $B_i=\frac{x_i+x_{i+1}+\cdots+x_{N+(i+1)}}{N}, i=1,\ldots,(n-N)-1$ *.*
Output*: Moving average of i.e.* $B=\left\{B_1,B_2,\ldots,B_{(n-N)-1}\right\}$ *.*

3.2 Non-polynomial cubic spline

Suppose Δ is a mesh on $[a,b]$ such that $\Delta : a=x_0<x_1<x_2<\cdots<x_{N-1}<x_N=b$ and $\{f_i\}$, $f_i=f(x_i)$, $i=1,2,\ldots,N$ are corresponding data points. Let $h_i=x_i-x_{i-1}, i=1,2,\ldots,N$ be the mesh size and mesh ratio be $\sigma=\frac{h_{i+1}}{h_i}$, $i=1,\ldots,N-1$. When $\sigma=1$, the mesh transforms to a uniform mesh, i.e., $h_{j+1}=h_j=h$. The non-polynomial $S_\Delta(x)$ is a function of class $C^2[a,b]$ has the following form:

$$S_{\Delta_i}(x)=a_i+b_i(x-x_i)+c_i\sin\tau(x-x_i)+d_i\cos\tau(x-x_i), \Delta_i=[x_i,x_{i+1}], \tag{2}$$

such that a_i,b_i,c_i and d_i are constants and τ is arbitrary parameter. Besides, τ is the frequency of the trigonometric part of the spline functions which has two important features as [18]:
First of all: τ is real or pure imaginary.
Secondly: τ is a factor can be used in order to improve the accuracy of method.

Assume u_i denotes approximation of $u(x_i)$ which obtained by segmentation non-polynomial spline function go through the points (x_i,u_i) and (x_{i+1},u_{i+1}). To capture essential conditions for computing coefficients in using interpolation conditions at x_i and x_{i+1} points that satisfies in them is not sufficient. Therefore, we have to apply the continuity of first derivative at the nodes (x_i,u_i) to compensate this shortage.

As a result of the non-polynomial cubic spline features, following expressions are derived as follows:

$$\begin{aligned} &S_\Delta(x_i)=u_i, \quad S_\Delta(x_{i+1})=u_{i+1},\\ &S''_\Delta(x_i)=M_i, \quad S''_\Delta(x_{i+1})=M_{i+1}. \end{aligned} \tag{3}$$

The coefficients of (2) are obtained by algebraic calculation:

$$\begin{aligned} a_i &= u_i + \tfrac{M_i}{\tau^2}, \quad b_i = \tfrac{u_{i+1}-u_i}{h} + \tfrac{M_{i+1}-M_i}{\tau\theta}, \\ c_i &= \tfrac{M_i\cos\theta - M_{i+1}}{\tau^2 \sin\theta}, \quad d_i = -\tfrac{M_i}{\tau^2}, \end{aligned} \tag{4}$$

where $\theta = \tau h$ and $i = 0,1,\ldots,N-1$. By applying the continuity of the first derivative at (x_i, u_i), i.e.,

$$S'_\Delta\left(x_i^-\right) = S'_\Delta\left(x_i^+\right), \tag{5}$$

the following expression is obtained

$$\alpha M_{i+1} + 2\beta M_i + \alpha M_{i-1} = \tfrac{1}{h^2}\left(u_{i+1} - 2u_i + u_{i-1}\right), \tag{6}$$

where $i = 0,1,\ldots,N-1$ and $\alpha = (\theta\csc\theta - 1)/h^2$, $\beta = (1 - \theta\cot\theta)/h^2$.

Applying the operator $E = e^{hD}$ in relation (6), we obtain

$$\begin{aligned} Error = (2\alpha + 2\beta - 1)\left(M_i - y_i''\right) + D^2h^2\left(\alpha - \tfrac{1}{12}\right)\left(M_i - y_i''\right) \\ + D^4h^4\left(\tfrac{\alpha}{12} - \tfrac{1}{360}\right)\left(M_i - y_i''\right) + O\left(h^6\right). \end{aligned} \tag{7}$$

The consistency relation (6) leads that the equation $2\alpha + 2\beta = 1$ in the relation (7) is satisfied, which can be expressed as $\tan\frac{\theta}{2} = \frac{\theta}{2}$. This equation has a zero root and infinitely non-zero roots, and $\theta = 8.98881$ is the smallest positive root. In this paper, this θ is considered as optimal value in all steps. By applying this assumption, we have

$$\left|M_l - y_l''\right| \le d_2 h^2,\ d_2 = 0.22\max\left|y_l^4\right|. \tag{8}$$

Assuming $\alpha = \frac{1}{12}$ and $\beta = \frac{5}{12}$ the equation $2\alpha + 2\beta = 1$ is satisfied. Therefore, the second term in (7) is zero. This choice leads to modify the order of the method, i.e., $O\left(h^4\right)$.

Besides, if natural spline initial condition, i.e., $M_0 = M_n = 0$ is considered as boundary equations, the system (6) is strictly diagonally dominant. Therefore, $M_1, \ldots, M_{n-1}$ are determined uniquely. The matrix form of system (6) is of the form

$$M = \tfrac{12}{h^2} N^{-1} JY. \tag{9}$$

where

$$M = \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ \vdots \\ \vdots \\ M_{n-2} \\ M_{n-1} \\ M_n \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 1 & 10 & 1 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & 10 & 1 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 1 & 10 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 10 & 1 \end{bmatrix}, \quad JY = \begin{bmatrix} 0 \\ y_0 - 2y_1 + y_2 \\ y_1 - 2y_2 + y_3 \\ \vdots \\ \vdots \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \\ 0 \end{bmatrix}.$$

The presented nonpolynomial cubic spline satisfies the following relation:

$$T_3 = \text{Span}\{1, x, \cos\tau x, \sin\tau x\}.$$

By using the Maclaurin series expansions, we have

$$T_3 = \text{Span}\left\{1, x, \tfrac{2}{\tau^2}(1-\cos\tau x), \tfrac{6}{\tau^3}(\tau x - \sin\tau x)\right\}. \tag{10}$$

Whenever $\tau \to 0$, we have

$$\lim_{\tau\to 0} \frac{2}{\tau^2}(\cos\tau x - 1) = x^2,$$
$$\lim_{\tau\to 0} \frac{6}{\tau^3}(\sin\tau x - \tau x) = x^3.$$

Therefore by using the above limits in the equation (10) we get $T_3 = \{1, x, x^2, x^3\}$. So that the nonpolynomial cubic spline reduces to the cubic spline when $\tau \to 0$ and then the relation (6) reduces to the consistency relation of the cubic spline

$$M_{i+1} + 4M_i + M_{i-1} = \frac{6}{h^2}(u_{i+1} - u_i + u_{i-1}). \tag{11}$$

According to above interpretation, convergence of non-polynomial cubic spline is concluded by Atkinsons logic [19] for natural cubic spline. It is worth noting, this method presents acceptable result when

$$h \le a \times 10^{-3}, \quad a > 0. \tag{12}$$

4 Evaluation metrics

Suggested methods' efficiency for characterization of ECG signal should be evaluated. Let $y(n)$ be the original signal and $x(n)$ be reconstructed ECG signal of length N. Besides, e denotes absolute error between original signal data and reconstructed signal. Some criteria for this purpose are defined as follows.

4.1 Root mean square difference

The root-mean-square difference (RMSD) evaluates average of the absolute error between main signal and approximated one:

$$RMSD = \sqrt{\frac{\sum_{i=1}^{N}[e(n)]^2}{N}}.$$

4.2 Percentage Root-Mean-Square Difference

The percentage root-mean-square difference (PRD) is used for calculation deformation in approximated signal as follows

$$PRD = \sqrt{\sum_{i=1}^{N}[y(n) - \bar{y}]^2},$$

where $\bar{y}$ denotes the average of the original signal.

4.3 Correlation coefficient

Correlation Coefficient (CC) is a criterion that specifies the degree such that original signal data and reconstructed signal are connected according to their morphology

$$CC = \frac{N\sum xy - \sum x \sum y}{\left[N\sum x^2 - \left(\sum x\right)^2\right]\left[N\sum y^2 - \left(\sum y\right)^2\right]}.$$

5 Numerical experiments

Evaluation of proposed method is done based on qualitative and quantitative analysis. Qualitative analysis discusses the efficiency of suggested method by visual evaluation. Besides, quantitative analysis is done based on criteria which is introduced in Section 4.

We use the following notations in the numerical results:
NCS: Non-polynomial cubic spline.
MAF: Moving Average Filter.
MNCS: Proposed algorithm in order to modification resolution of ECG signal based on Non-polynomial cubic spline.

Qualitative analysis

In order to survey the effect of proposed method, first of all, NCS method is implemented on four ECG samples which are displayed by (a), (b), (c) and (d) notation. In Fig. 3, implementation results of approximated signals are represented.

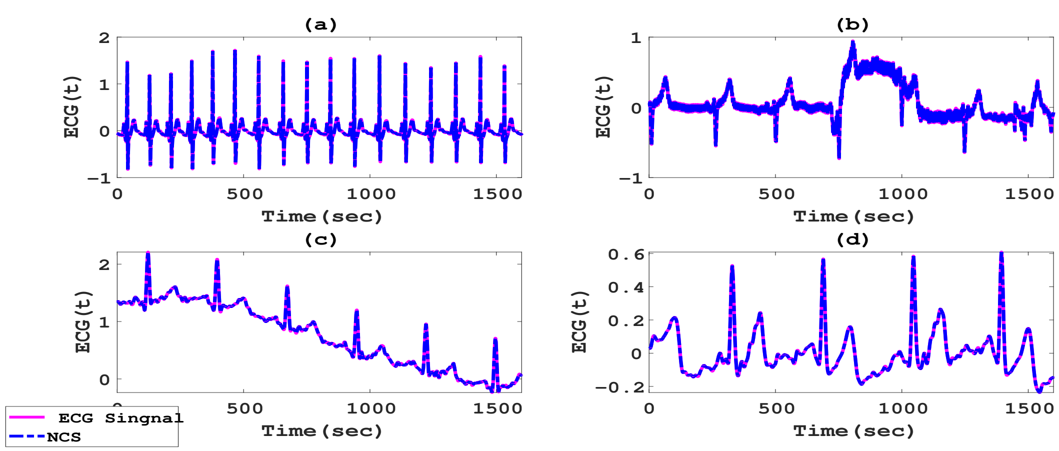

Figure 3: Approximation result of ECG signal by NCS method, (a) displays normal ECG sample and (b), (c) and (d) represent abnormal ECG signal samples.

According to Fig. 3, similarity between original ECG signal and its reconstructed form is specified. From a medical perspective, QRS complex consists important information about patients heart condition. Therefore, having accurate QRS complex plays a decisive factor in hearts disease. In order to better observation of results interpolation methods which are presented above, QRS complex of one complete wave of normal ECG is approximated by NCS method and the result is displayed in Fig. 4. According to Fig. 4, capability of this method in the field of reconstruction of ECG signal is visible especially in Fig. 4 (b).

For better representation of proposed methods efficiency in the field of approximation of signal, we add white Gaussian noise with SNR=5,10 dB to normal ECG Signal samples and approximated noisy ECG signal by using NCS method. In Fig. 5- 6, the result of implementing method on data is represented. As can be seen from Fig. 5-6 reconstructed noisy ECG signal are smoother compared with noisy ECG samples.

Quantitative analysis

Metrics which are introduced in section 4 are calculated in order to evaluation of proposed method's efficiency quantitatively. Although used metric criteria are explain in section 4 precisely, it is worth mentioning a few points

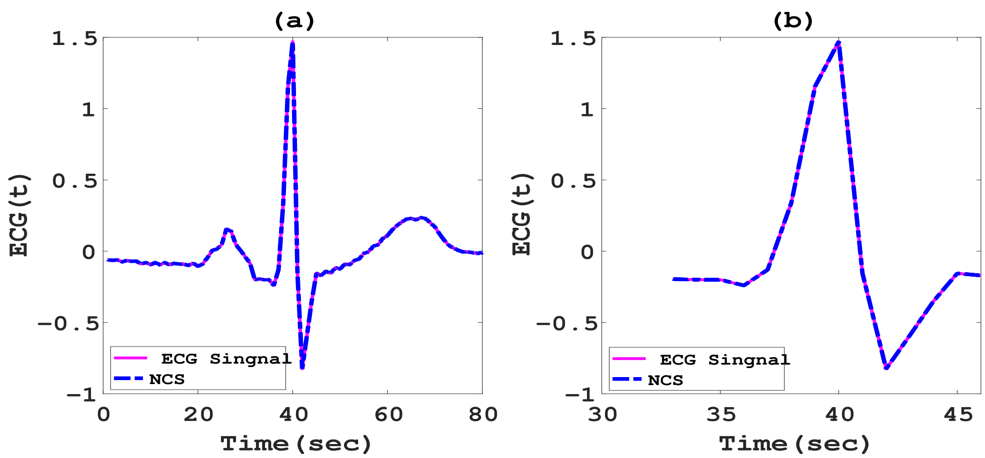

Figure 4: Approximation of QRS complex of one complete wave of ECG by NCS Method, (a) one complete wave of normal ECG signal that is approximated by NCS method (b) QRS complex of part (b) is focused.

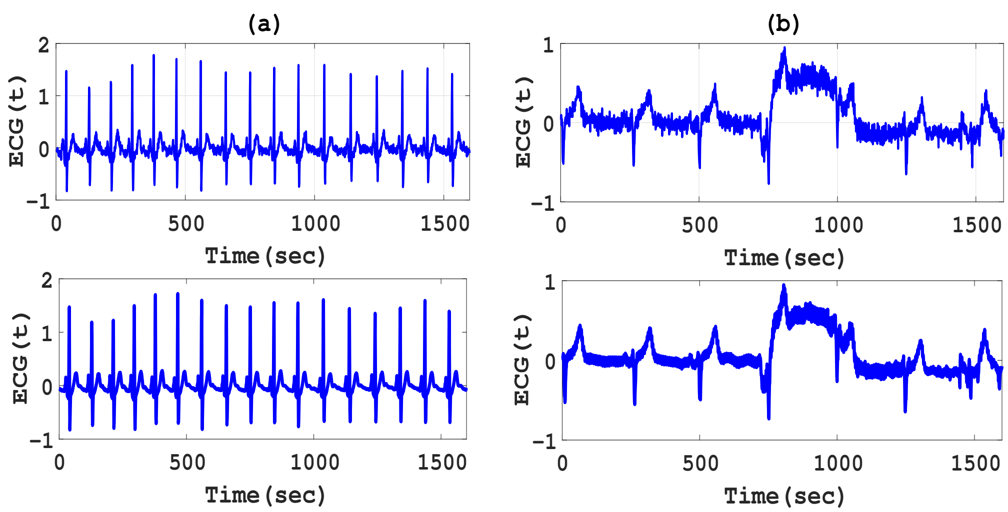

Figure 5: Comparison of noisy ECG signals with SNR= 10dB (top panel) and reconstructed ECG signal by MNCS method (bottom panels).

about acceptable values for these criteria. RMSD is always non-negative, zero (almost never attained in practice) value represents a complete fit to the data. Besides, the quality of the approximated signal is evaluated as good approximation if the PRD (%) scale is between 0 and 9. Moreover, CC which is a criterion for similarity between original signal and approximated one is close to 1. The results of implementing proposed method is represented in Table 1.

As can be seen from Table 1, the results of RMSD and PRD are small acceptably for all of samples. Besides, CC for each sample is close to 1 sufficiently which displays the similarity between original ECG sample and its reconstructed one. Therefore, the proposed method is capable in order to approximate the ECG samples.

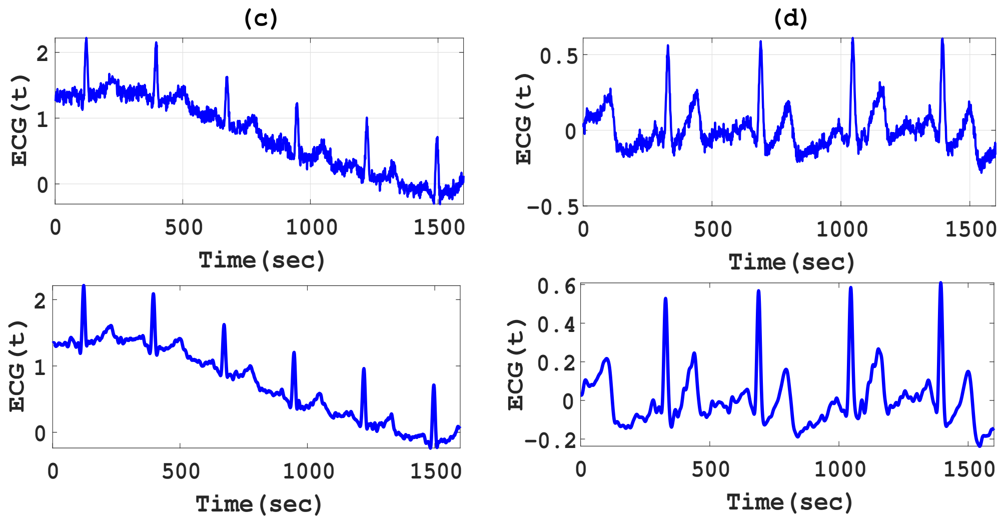

Figure 6: Comparison of noisy ECG signals with SNR= 10 dB (top panel) and reconstructed ECG signal by MNCS method (bottom panels).

Table 1: Numerical results of Samples (a)-(d) Based on Metric criteria (*RMSE*, *PRD* and *CC*).

N	sample (a)			sample (b)			sample (c)			sample (d)		
	RMSD	PRD	CC	RMSD	PRD	CC	RMSD	PRD	CC	RMSD	PRD	CC
100	0.00400	2.28751	0.99396	0.00516	1.37747	0.99997	0.00500	1.65745	0.99574	0.00500	1.28751	0.99996
110	0.00401	2.55683	0.99487	0.00769	1.36623	0.99996	0.00247	1.99647	0.99425	0.00501	1.55683	0.99487
120	0.00457	2.69534	0.99523	0.00556	1.07069	0.99997	0.00287	1.43049	0.99743	0.00557	1.69534	0.99923
130	0.00426	1.48142	0.99937	0.00580	1.20547	0.99998	0.00221	1.49650	0.99164	0.00426	1.48142	0.99937
140	0.00666	2.23612	0.99776	0.00813	1.25624	0.99996	0.00169	2.05476	0.99624	0.00466	1.23612	0.99976
150	0.00557	3.29116	0.99262	0.00227	1.21244	0.99998	0.00922	1.45354	0.99617	0.00457	1.29116	0.99962
160	0.00402	2.97304	0.99318	0.00238	1.30797	0.99996	0.00978	2.40898	0.99492	0.00392	1.97304	0.99918
170	0.00609	2.56799	0.99447	0.00784	1.29649	0.99996	0.00227	2.67953	0.99222	0.00300	1.56799	0.99947
180	0.00274	2.28824	0.99231	0.00988	1.16276	0.99996	0.00252	2.24812	0.99518	0.00239	1.28824	0.99931
190	0.00239	2.64631	0.99564	0.00776	1.21834	0.99999	0.00402	1.58782	0.99305	0.00250	1.64631	0.99964
200	0.00250	2.72755	0.99430	0.00757	1.05270	0.99996	0.00180	1.99831	0.99186	0.00250	1.72755	0.99990
210	0.00289	2.18217	0.99316	0.00669	1.43816	0.99996	0.00083	1.18431	0.99242	0.00289	1.78217	0.99916

The numerical result of proposed method and [27] on sample(d) are illustrated in Table 2. the order of Lagrange-Chebyshev restoration is order 50. According to the numerical result of Table 2, the accuracy of proposed method is clearly visible. In terms of elapsed CPU time, the proposed method is superior to method of [27].

6 Conclusion

From a medical point of view, precise characterization of ECG signals was considered as a decisive factor in accurate diagnosis of the disease. MNCS algorithm was proposed as a mathematical model in order to modify the ECG signal quality in resolution. Capability of this method was evaluated by simulation result and quantitative criteria. Therefore, MNCS method represented acceptable results diagnostically. As can be seen, these methods can be used in similar fields in order to reconstruct the signals.

Table 2: Quality assessment metrics for proposed method and [27].

	sample (d)				[27]			
N	RMSD	PRD	CC	CPU Time	RMSD	PRD	CC	CPU Time
250	0.00530	1.58051	0.99996	0.99999	0.06363	2.35671	0.98897	2.00011
500	0.00491	1.56783	0.99586	1.00011	0.05132	2.56412	0.97012	2.69871
1000	0.00546	1.66014	0.99961	1.45672	0.06897	2.76903	0.97193	2.99989
1250	0.00536	1.68602	0.99987	2.00011	0.07124	2.88806	0.97119	3.50014
1500	0.00394	1.03452	0.99986	2.45610	0.08666	2.67015	0.96456	4.00983
1750	0.00367	1.07816	0.99982	3.0012	0.05696	2.29940	0.98003	5.57780
2000	0.00332	1.99994	0.99918	3.50631	0.06145	2.90237	0.98636	5.45875

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