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A simulated annealing algorithm for the restricted stochastic traveling salesman problem with exponentially distributed arc lengths

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Abstract. The considered stochastic travelling salesman problem is defined where the costs are distributed exponentially. The costs are symmetric and they satisfy the triangular inequality. A discrete time Markov chain is established in some periods of time. A stochastic tour is created in a dynamic recursive way and the best node is detected to traverse in each period. Then, a simulated annealing based heuristic method is applied to select the best state. All the nodes should be traversed exactly once. An initial ρ-approximate solution is applied for some benchmark problems and the obtained solutions are improved by a simulated annealing heuristic method.

Keywords: Travelling salesman problem, discrete time Markov chain, approximation algorithms, simulated annealing.
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1 Introduction

The symmetric traveling salesman problem (S-TSP) is known as NP-hard problem [6, 17], where the costs for the forward and the backwark direc-

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tions are symetrically equal [2]. However, the other version of the problem that the forward and the backward costs are not equal is known as the asymmetric traveling salesman problem [21]. There are wide variety of applications for traveling salesman problems, e.g. transportation and telecommunication networks, manufacturing [3,5,6]. Zhang et al. [27] considered some intermediate destination nodes and they are assumed to be traversed at least once in the optimal solution. The quota traveling salesman problem (Q-TSP) was presented by Ausiello et al. [3] and the tour is restricted to reach a given quota in minimum time. Sun et al. [24] considered some periods for customer nodes to be visited several times. Jaillet and Lu [12] assumed some requests and penalties for not serving ones; they applied some advanced information to produce their algorithms. Ausiello et al. [4] considered two versions of the asymmetric traveling salesman problem where the distance function does not satisfy the symmetry condition. Yu et al. [26] concerned some online variant of the Q-TSP in the asymmetric spaces. Wen et al. [25] considered a general metric space where requests have a release time and a deadline.

Delay is an important parameter of some various applications, especially in transportation and telecommunication networks that it causes flow congestion and consequently hazardous collapse of the networks. So, the times on the arcs are interpreted as the delay to traverse any arc, where the arc delays could be unknown and varying over time. Kalaia and Vempala [13] considered the user is informed on the times on all arcs when a path between given source and destination nodes is selected by the user. Also, the congestion in the networks is distributed exponentially over the time parameter, so, some stochastic routing models are introduced [1,20]. The stochastic traversal arc lengths are assumed by Provan [20], where the lengths become known upon arrival at the tail nodes; then Ardakani and Sun [1] applied the stochastic realization of the arc costs.

In our considered stochastic networks, decisions are made dynamically in some periods of time, and the best node should be determined to traverse in each period. The arc lengths are distributed exponentially, and then, a discrete time Markov chain (DTMC) is established in an undirected network, in some periods of time. Thus, the states of the DTMC are feasible tours such that any node of the network is restricted to traverse exactly once. Let $G = (N, A)$ be a complete undirected network with node set N and arc set A. For any $(i, j) \in A$, c_{ij} is the cost parameter from node j toward i, and $c_{ij} = c_{ji}$ (symmetry), also $c_{ij} \leq c_{ik} + c_{kj}$ for any $(i, j) \in A$ and $k \in N$ (triangular inequality). In the considered stochastic network, the arc costs are distributed exponentially, so they are the expected values (the exponential distribution parameters); also, decisions are made dynamically in some periods of time, and the best node should be determined to traverse in each period.

Kalaia and Vempala [13] modeled the times on the all arcs are revealed when a path from a given source toward a given destination by the user. Provan [20] considered the stochastic traversal arc lengths those become known upon arrival at the tail nodes. Ardakani and Sun [1] applied the stochastic realization of the arc costs. Awerbuch and Kleinberg [5] considered a network with unknown arc delays varying over time.

For such an NP-hard problem, where the arc costs satisfy the triangular inequality, there are some approximation algorithms [7, 10]. However, the algorithms based on the simulated annealing (SA) are one of the most applied methods to solve the routing problems [8,9,14]. Then, we proposed a simulated annealing (SA) based heuristic algorithm. So, the obtained solution is the most improved one in this setting, and it is resulted by traversing nodes exactly once dynamically over some periods of time.

Christofides [6] proposed an approximation algorithm where the triangular inequality is satisfied by the cost parameters. So, the triangular inequality is an essential assumption to produce an approximation solution (see [6, 7]); then, it was used by [3, 4, 12, 25–27] and also by this work. At some periods of time, the next node is selected dynamically by starting from the given source node, and the source node will be returned when all nodes are traversed exactly once. The dynamic routing policies determine the order of the nodes. The decisions are made according to the established DTMC stochastic process.

Kalaia and Vempala [13] defined some decision periods, and in each period they chose the best decision. In the model by Ausiello et al. [3] and Wen et al. [25], a given number of nodes are traversed and not necessarily all the nodes. However, there are some models that a node is allowed to visit several times [4, 27]. Jaillet and Lu [12] assumed some penalties for not served destination nodes. Thus, in the proposed model, all nodes are traversed exactly once (except the source node).

The paper is organized as follows. Section 2 describes the established DTMC in the network and its constructed state space. Section 3 presents a meta-heuristic method based on SA and the structure of the DTMC. Section 4 contains some numerical results on some well-known bench-mark instances.

2 The established discrete time Markov chain

In this section, the proposed method by Shirdel and Abdolhosseinzadeh [23] is presented to establish a DTMC at any period of time, when it should be decided which node is the next one, dynamically. Initially, the source node is traversed and the process is finished by traversing all nodes exactly once, eventually. It was previously established a DTMC for the stochastic shortest path problem [22].

The state $S_{t,k}$, $t = 1, \dots, n-2$ and $k = 1, 2, \dots, n-(t+1)$ contains those nodes do not create a necessarily optimal tour. The pre-obtained approximate solution is considered as the initial state and its first node is fixed (the source node). A fixed node was traversed previously at the current policy time t. At any time, only one node is traversed, and then it will be fixed. So, the next state $S_{t+1,j}$ is created by single permutation of the unfixed nodes in the current state $S^*_{t,i}$. So, if v_{k-1} is the last fixed node (the fixed nodes are shown by a bar accent) of state $S^*_{t,i}$=$\{1$=$\overline{v_1}, \overline{v_2}, \dots, \overline{v_{k-1}}, v_k, v_{k+1}, \dots, v_n, 1\}$, then state S_{t+1j} is created by permutation node v_k with unfixed nodes v_{k+1}, v_{k+2}, $\dots$, v_n ($|N| = n$). The total size of the general search space for network G with n nodes is $(n-2)\,(n-1)/2$+1. The created state space is produced by the following principles:

(i) The initial state $S_{0,1}$ contains the pre-obtained approximate solution and its first node is fixed.

(ii) There is not any repeated state among those are created by an unfixed index. Clearly, any state could be taken account into its opposite direction and both are the same. The source node is the first and the last node of the created states, so the permutation is a circular permutation.

(iii) If there are at least two fixed indices, the permutation does not cause a repeated state because it is not possible double (or more) permutation and just a single node allowed to be permuted.

(iv) To determine the repeated states consider the created states starting from the initial state. The repeated state cannot occur after index $2n-5$ (double permutation is not possible), however it may be occurred before the following way:

   (a) The initial state could be repeated in index $n-2$, so $v_{n-1} = v_3$ then
   $\{1 = \overline{v_1}, v_2, v_3, \dots, v_{n-1}, v_n, 1\} \equiv \{1 = \overline{v_1}, v_n, v_3, \dots, v_{n-1}, v_2, 1\}$
   $\Rightarrow n-1 = 3 \Rightarrow n = 4$.

(b) The state of index $n-2$ (where v_{k_2} is unfixed) could be repeated through the states from $n-1$ to $2n-5$ (where $\overline{v_{k_2}}$ is fixed). Suppose index r is a repeated state $\{1=\overline{v_1}, v_n, v_3, \ldots, v_{n-1}, v_2, 1\} \equiv \{1=\overline{v_{l_1}}, \overline{v_{l_2}}, v_{l_3}, \ldots, v_{l_{n-2}}, v_{l_{n-1}}, v_{l_n}, 1\}$, then $\overline{v_{l_1}} = \overline{v_1} = 1$,$v_{l_{n-1}} = v_3$ and $v_{l_n} = v_2$, so if $v_4 = \emptyset$ then $l_{n-2} = \emptyset$ and $n = 5$, otherwise if $v_4 \neq \emptyset$ then $v_{l_{n-2}} = v_4$ and $n = 6$.

Then, there is no repeated state in the created states of the dynamic established DTMC for $n \geq 7$.

Let $I_{S^*_{t,i}}$ be the unfixed nodes of state $S^*_{t,i}$, then the transition probabilities $p_{i,j}$, from state $S^*_{t,i}$ toward state $S_{t+1,j}$, are defined exponentially as follows

$$P_{i,j} = \begin{cases} \frac{1}{c_{ij}} e^{-\frac{1}{c_{ij}}}, & \text{if } j \in I_{S^*_{t,i}}, \\ 0, & \text{if } j \notin I_{S^*_{t,i}}, \end{cases}$$

where c_{ij} is the expected cost of arc $(i,j) \in A$ and $j \in I_{S^*_{t,i}}$, and t is the current period of time. Also, there is $p_{i,j} = p_{j,i}$ because of symmetry. The initial state is improved directly in the established DTMC with respect to the dynamically made decisions. However, Provan [20] proposed a Markov process that its states are created by the realizations of the arc costs.

3 The simulated annealing heuristic

The initial state $S^*_{0,1}$ shows a ρ-approximate solution. For example, there is a 2-approximate solution by minimum spanning tree (see [17]), and Christofides [6] produced a 3/2-approximation. The next possible nodes are determined by the possible transitions from the current state toward the next states.

Markov decision problems are solved in polynomial time [17], but computations grow exponentially in practice [15]. Then, the Markov chain formulation of the SA [19] is applied to solve the problem.

Let $\overline{C}_{S_{t,j}}(T)$ be the expected cost of state $S_{t,j}$ in temperature T, then $\triangle\overline{C}_{i,j}(T) = \overline{C}_{S_{t+1,j}}(T) - \overline{C}_{S^*_{t,i}}(T)$ is the expected cost difference of the transition from state $S^*_{t,i}$ toward state $S_{t+1,j}$. The acceptance probability $A_{i,j}(T)$ determines the probability of the accepting $S_{t+1,j}$ when it is currently in $S^*_{t,i}$. The acceptance probabilities are defined by Metropolis rule as follows

$$A_{i,j}(T) = \begin{cases} e^{\frac{-\Delta\overline{C}_{i,j}(T)}{T}}, & \text{if } \Delta\overline{C}_{i,j}(T) > 0, \\ 0, & \text{if } \Delta\overline{C}_{i,j}(T) \leq 0. \end{cases}$$

In the case $\Delta\overline{C}_{i,j}(T) > 0$, the acceptance of state $S_{t+1,j}$ is decided by producing a random number r; so if r satisfies the inequality

$$r \leq e^{-}\Delta\overline{C}_{i,j}(T)/T,$$

then $S_{t+1,j}$ is accepted and otherwise it is rejected.

The transition probability $M_{i,j}(T)$ from state $S^*_{t,i}$ toward state $S_{t+1,j}$ is obtained as follows

$$M_{i,j}(T) = \begin{cases} p_{i,j}A_{i,j}(T), & \text{if } i \neq j, \\ 1 - \sum_{k\neq i} p_{i,k}A_{k,j}(T), & \text{if } i = j. \end{cases}$$

The static probability $q_{i,j}(T)$ represents the probability that the policy determines to be in state $S_{t+1,j}$, when it is currently in $S^*_{t,i}$. The steady state analysis for the established DTMC determines the limiting-state probabilities that show the equivalent static [11].

Theorem 1. *The static probabilities when the SA is in the equilibrium state are computed by the following recursive equations*

$$q_{1,i}(T) = \frac{1}{1+\sum_{k\in I_{S^*_{t,i}}}\frac{A_{1,k}(T)}{A_{k,1}(T)}}, \quad q_{j,i}(T) = \frac{A_{1,j}(T)}{A_{j,1}(T)}q_{1,i}(T), \quad j \in I_{S^*_{t,i}}.$$

Proof. To simplify, let $|I_{S^*_{t,i}}| = R$ and T is omitted from the formulas, also $p_{1,k} = p_{1,k} = p_k$ and $q_{k,i} = q_k$ for $k = 2, 3, \ldots, R$. By the limiting state linear system of equations for DTMC (see [11]) we have

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_R \end{bmatrix}^{tr} \begin{bmatrix} 1-\sum_{k=2}^{R} p_kA_{1,k} & p_2A_{1,2} & p_3A_{1,3} & \ldots & p_RA_{1,R} \\ p_2A_{2,1} & 1-p_2A_{2,1} & 0 & \ldots & 0 \\ p_3A_{3,1} & 0 & 1-p_3A_{3,1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_RA_{R,1} & 0 & 0 & \ldots & 1-p_RA_{R,1} \end{bmatrix} = \begin{bmatrix} q_1, q_2, q_3, \ldots, q_R \end{bmatrix}.$$

By the last $R-1$ equations we have

$$\begin{cases} q_1p_2A_{1,2} + q_2(1-p_2A_{2,1}) = q_2, \\ q_1p_3A_{1,3} + q_3(1-p_3A_{3,1}) = q_3, \\ \qquad\vdots \\ q_1p_RA_{1,R} + q_R(1-p_RA_{R,1}) = q_R, \end{cases}$$

then $q_2 = \frac{A_{1,2}}{A_{2,1}} q_1$, $q_3 = \frac{A_{1,3}}{A_{3,1}} q_1$, …, $q_R = \frac{A_{1,R}}{A_{R,1}} q_1$. By the Gaussian elimination method on the first row, we get

$$q_1 + q_2 + q_3 + \cdots + q_R = 1 \Rightarrow q_1 = \frac{1}{1 + \sum_{k=2}^{R} \frac{A_{1,k}}{A_{k,1}}},$$

which completes the proof. □

The SA starts with an enough high initial temperature, such that all of the states are accepted. Then, the temperature is set practically to traverse all nodes. The annealing parameters are tuned by the proposed method [18]. If there is no state with $\triangle \overline{C}_{i,j}(T) \leq 0$, then the temperature is decreased by the rate $\alpha = T_0^{-1/(n-3)}$, where T_0 is the initial temperature and so $T := \alpha T$. Nourani and Andresen [16] showed the constant cooling system (as we applied) is the best against exponential and logarithmic ones. It is started with a high temperature and then it decreases the temperature by a constant factor, repeatedly.

4 Experimental results

We applied our method on some benchmark problems provided by TSPLIB[1] those are some instance networks with Euclidean distances, and their optimal solutions are given: a280, berlin52, ch130, ch150, eil51, eil76, eil101, kroA100, kroC100, kroD100, lin105, pcb442, pr76, pr1002, pr2392, rd100, st70, tsp225. The arc costs are obtained by 2D Euclidean distance function. The given distances are considered as the exponential distribution parameter $(1/c_{ij})$.

Ausiello et al. [3] obtained ratio 2 where all nodes are not traversed, necessarily. Jaillet and Lu [12] applied advanced information and presented ratio 2 for the real line, ratio 2.28 for the general metric space and also ratio $1.5\rho+1$ by any ρ-approximation algorithm. Ausiello et al. [?] gave ratio $\frac{3+\sqrt{5}}{2}$ for the best possible implementation. Yu et al. [26] obtained ratio $1 + \rho$ and ρ is an approximation ratio. Zhang et al. [27] presented ratios $1 + k$ and $4 + k$ lower bounds and k is the number of blockage arcs. Wen et al. [25] obtained ratio $2L + 1$ for the line segment $[-L, L]$ and 2 where every arc has unit weight.

The instances and some obtained results by the initial approximate solution and also by the initial random solution are shown in Table 1. The

[1]Ruprecht-Karls-Universität Heidelberg, http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/index.html.

Table 1: The obtained results for the instance networks.

network	node numbers	initial ratio	Improved ratio	Improved ratio (reverse)	random initial solution improvement %	random initial solution improvement (reverse) %
a280	280	1.3472	1.3111	1.3007	45.7004	48.4242
berlin52	52	1.3790	1.2249	1.1417	45.4408	38.1485
ch130	130	1.3544	1.2881	1.2456	52.3719	38.3771
ch150	150	1.4088	1.3373	1.2625	50.7428	42.2111
eil51	51	1.4311	1.3628	1.3202	38.6569	39.7421
eil76	76	1.3738	1.3046	1.2995	39.8537	38.6260
eil101	101	1.3403	1.2770	1.2309	37.8530	38.4776
kroA100	100	1.4337	1.3731	1.3554	46.7892	44.1295
kroC100	100	1.3317	1.2479	1.2136	44.7716	48.2430
kroD100	100	1.3430	1.2660	1.2755	44.3940	42.1245
lin105	105	1.4719	1.3950	1.3849	50.0172	38.1500
pcb442	442	1.4036	1.3359	1.3322	59.0397	46.5664
pr76	76	1.3653	1.3309	1.3077	52.1535	44.1995
pr1002	1002	1.3565	1.2869	1.2745	65.5310	51.5144
pr2392	2392	1.3970	1.3047	1.3024	63.1005	53.0013
rd100	100	1.3232	1.2667	1.2560	52.4740	42.4809
st70	70	1.2699	1.1925	1.1887	44.2053	46.2880
tsp225	225	1.3990	1.3211	1.3104	47.1672	45.3409

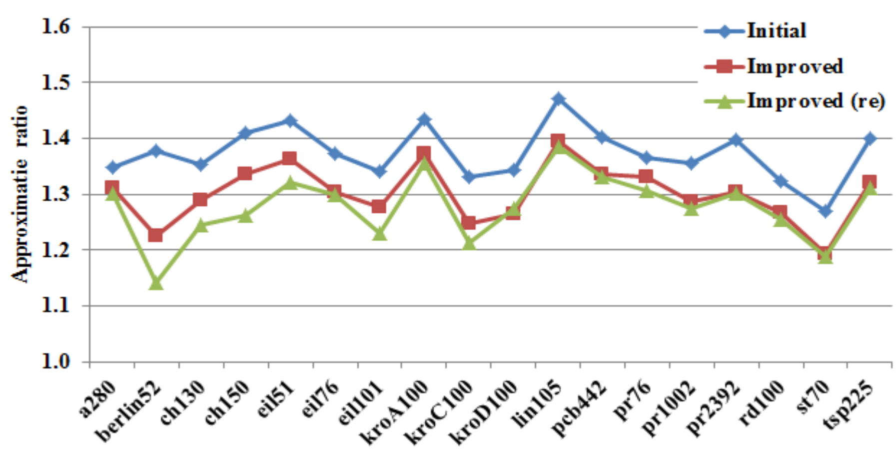

Figure 1: The improvements of the initial approximate solutions for some instances.

initial approximate solution is computed by the Christofides' algorithm [6] that provides a 3/2-approximation solution. The SA parameters are determined experimentally. The fixed (traversed) nodes' order is not changed by permutation and every node is traverse exactly once. So, the initial approximate tour is considered in the reverse order and the algorithm is implemented again. The performance of the obtained solution is computed as the approximation ratio, and the improvement shows the percentage (initial solution − improved solution)/initial solution × 100.

The improved approximate ratios are shown in Figure 1, where "Improved (re)" is associated to the obtained results for the reverse order of the initial solution. So, the obtained approximation ratio is 1.3 for the initial solution and it is 1.28 for the initial reverse order solution, in average.

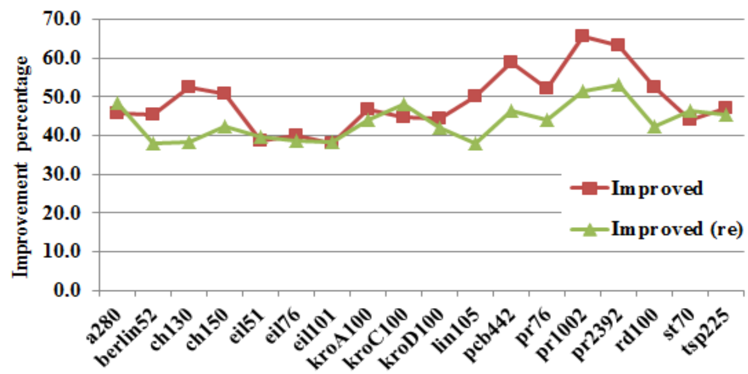

Figure 2: The improvements of the initial random solutions for some instances

To show the improving capability of the algorithm, the input is given by an arbitrary random initial solution during the restricted implementation numbers of the algorithm. The improvement capability of the proposed solution method for initial arbitrary random solutions is shown in Figure 2, where it is about 49% and 44% improvements for the initial solution and its reverse order, in average.

5 Conclusion

The symmetric traveling salesman problem was considered, however it is restricted to traverse all nodes exactly once. A discrete time Markov chain with exponential distributions of the transition probabilities was established dynamically. Then, a simulated annealing heuristic method was applied to improve the initial feasible solution. The improvement of the approximation ratios were computed for the obtained tours in some benchmark problems. So, the results determined the proposed method improvement capability.

We set the dynamic policy times (state numbers) equal to the number of nodes, so the algorithm was terminated when all the nodes are traversed exactly once and it was returned to the source node, finally. Then, the created state spaces could imply a better quality of the approximate solutions. The other search methods, e.g., the genetic algorithms and the hybrid algorithms may lead to some better solutions.

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