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Modeling the spread dynamics of racism in cyberspace

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Abstract. In this work, we develop a new $SEID$ (Susceptible-Exposed-Infected-Deny) racism propagation model, which describes the racism diffusion. The racism-free and prevalence equilibrium points are calculated and its stability is analyzed. The threshold value of the model R_0 is derived. As a result of theoretical analysis, racism spread is under control when $R_0 \leq 1$. While, if $R_0 > 1$ the racism propagates in the cyberspace. Furthermore, the sensitivity analysis of the parametric values of the model are illustrated. We use Matlab ode45 solver to illustrate the numerical solutions. Finally, from theoretical analysis and numerical solutions, we obtain mutually consistent results.

Keywords: Cyberspace, racism spread, compartment model, stability analysis, numerical results.
AMS Subject Classification 2010: 34A34, 65L12, 68M11.

1 Introduction

Compound multicultural societies hold together through effective and cooperative communication, which strengthens civility, enrich information sharing, and enables the expression of interests while allowing both diversity and cohesion [17, 18].

The Internet, which has become an essential platform of the communication systems in the current societies, offers both merits and challenges.

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Nowadays, internet clients mainly use electronic device communication applications for delivering breaking news or recent messages, such as Facebook, Twitter, video-sharing platform YouTube, etc. [5, 17]. In 21-century information technology, social network applications are connected billions of people in the cyberspace [6, 22]. We receive news online timely, we chat with friends via social media, and enhance our skills through online professional cyberspace. However, there are numerous cruel sides of the internet that far-right users exploit to spread conflict-ridden ideas, racial hate, and mistrust. This attacks social cohesion and aims to carry out singular, and exclusive racial, ethnic or religious social norms. Researchers of racism refer to this type of racist communication online as cyber-racism [11, 16].

In the current world propagation of racism through Web 2.0 technologies bring the worst damage to the social life of human beings. The prevalent spread of racist information can principal to a series of serious risks, such as social uncertainty, large business distraction, and a decline in the social consistency of modern multicultural societies. Political activities can affect to establish or re-establish preferential treatment based on race or exploit racist sentiment for political purposes use information technology to propagate their messages. Such calls have claimed their power to affect every societal legacies and influence on election results. So, to study the propagation process and recommend the control measure of such real-life problems is timely extensive research potential [2, 3, 11].

Mathematical modeling has a mimic role to study the system dynamics of real-world problems in different disciplines. It has been used extensively, in ecological and epidemiological sciences, see reference [1, 4, 8, 10, 12, 15]. The idea of system modeling can be adopted in the social aspect, especially social behavior dynamism. This modeling works are develop cyber-based information flow through internet platforms [13, 14, 20, 23]. However, this research works can't address racism dynamics in social networks. It has one of a series issue in multicultural society based countries like Ethiopia [9], its affect daily life of innocent people.

An effective mathematical model for racism spreading will have major theoretical significance to understand the dissemination dynamics and resilience of the spreading processes, and then propose practical measures for minimizing damages caused by racism. The model can characterize and analyze the critical threshold of racism spreading in cyberspace. Study of racism spreading is primarily based on either model of opinion dynamics or upon models of biological epidemics [14, 15]. In our study, we take a compartmental approach, similar to some of the systems models that have been developed mainly to model the spread of disease. In our model, we consider the person as user nodes, the new users who come to cyberspace as recruitment(birth) and the one who has to deactivate the account is death. User node based studies of the spread of racism have led to major breakthroughs in our understanding of racism spread.

2 Model Formulation

In this paper, we consider the whole population $N(t)$ to actively use and to be located relevant cyberspace, and people can be represented as nodes at time t. In this work, we just aim to model cyber racism spreading over a fixed connected cyberspace. Our model is based on dividing the whole user population into four-compartment: S, E, I, and D. At any time t, each user adopts one of four compartments S, E, I, or D, that are shown in Figure 1 and described as

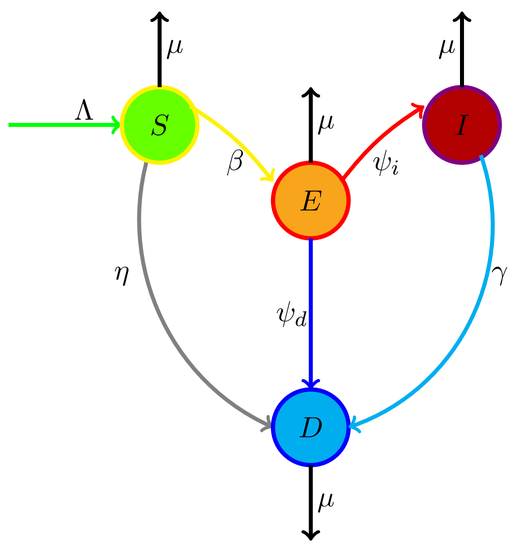

Figure 1: Schematic diagram for racism spreading model.

- Racism-Free individuals(nodes). The number of people who have never contacted with racism. These individuals mean that who have not yet received or heard the propagating racism message, while it can receive or hear the message, which we call it **Susceptible**. $S(t)$ denotes the number of nodes have not received or heard the racism message at time t.

- Racism-Exposed individuals(nodes). The people who have already received/heard the racism information from the racist state. These individuals may have been racist, while they are in a hesitate state and do not spread racism which we call it **Exposed**. $E(t)$ is used to denote the number of nodes having already received or heard the racism message yet not decided to activate it at a time t.

- Racism-Infected individuals(nodes). These people have received or heard information and believe that the information is true. They are choosing to like, comment, speech, write, share with others the racist information, and support the activity facially. These peoples mean that who are actively spreading the racism which we call it racially **Infected**. $I(t)$ is introduced to represent the number of nodes whose users choose to believe and spread it after reading or hearing the racism message through their cyberspace at time t(leave the incubation period and they decide to be activist).

- Racism-Denied individuals(nodes). The people who have known the racism but never believed and spread it or after hearing the racism information, they choose to deny it which we call it **Deniers**. $D(t)$ is introduced to denote the number of individuals who have read or hear the racism message and then choose to deny it at time t.

Here, we describe the basic assumptions and associated parameters value of the model.

1. An S-node is assumed to believe and ignore the racism message sent by a single I-node with the probability of β and η per unit time respectively. Racism propagation occurs when S-nodes can be connected to I-nodes at any time t.

2. An E-node users can be changed to I-node with the probability of ψ_i due to trust and interest in the racism and with the probability ψ_d it can be changed to D-node due to ignorance or deny of racism per unit time.

3. It is assumed that the users who believe the racism information are possible to forward the messages over their cyberspace, leading to racism prevalence. A racism-believed node will convert to be a racism-denied node due to different reasons with the probability of γ per unit time.

4. The parameter Λ is the recruitment of the cyberspace, while μ leaving the rate of each state nodes.

According to the above basic assumptions, the process of cyber racism dynamism over the fixed cyberspace is shown in Figure 1. The model can be described by the following system of nonlinear ordinary differential equations:

$$\begin{aligned} S'(t) &= \Lambda - (\beta+\eta)\frac{S(t)I(t)}{N} - \mu S(t), \\ E'(t) &= \frac{\beta S(t)I(t)}{N} - (\psi_d + \psi_i + \mu)E(t), \\ I'(t) &= \psi_i E(t) - (\gamma+\mu)I(t), \\ D'(t) &= \frac{\eta S(t)I(t)}{N} + \gamma I(t) + \psi_d E(t) - \mu D(t). \end{aligned} \tag{1}$$

We consider all parameter values non-negative.

For simplicity of our mathematical analysis, the variables of the model (1) can be scaled as

$u(t) = S(t)/N$, $v(t) = E(t)/N$, $w(t) = I(t)/N$, $d(t) = D(t)/N$and $\Lambda = \mu N$.

After substituting it in system(1), we can get the simplified system:

$$\begin{aligned} u'(t) &= \mu - (\beta+\eta)u(t)w(t) - \mu u(t), \\ v'(t) &= \beta u(t)w(t) - (\psi_d + \psi_i + \mu)v(t), \\ w'(t) &= \psi_i v(t) - (\gamma+\mu)w(t), \\ d'(t) &= \eta u(t)w(t) + \gamma w(t) + \psi_d v(t) - \mu d(t). \end{aligned} \tag{2}$$

The total population of the model is considered as a constant i.e.,

$$N(t) = S(t) + E(t) + I(t) + D(t).$$

According to normalization of model (1) we have an identity

$$u(t) + v(t) + d(t) + w(t) = 1.$$

Next, one equation of the system (2) can be eliminated and we hold triple system of differential equations

$$\begin{aligned} v'(t) &= \beta w(t)\left(1 - v(t) - d(t) - w(t)\right) - (\psi_d + \psi_i + \mu)v(t), \\ w'(t) &= \psi_i v(t) - (\gamma+\mu)w(t), \\ d'(t) &= \eta(1 - v(t) - d(t) - w(t))w(t) + \gamma w(t) + \psi_d v(t) - \mu d(t). \end{aligned} \tag{3}$$

So, the physical domain of the system (3) is

$$\Omega = \left\{(v, w, d) \in R_+^3 | v + w + d \leq 1\right\}.$$

For the well-posedness of the model, we have the following important lemma.

Lemma 1. *The set Ω is positively invariant to system (3).*

Proof. Denote $y(t) = (v(t), d(t), w(t))^T$ and then system (3) can be rewritten as

$$\frac{dy(t)}{dt} = f(y(t)),$$

where

$$\begin{aligned} f(y(t)) = [&\beta w(t)\,(1 - v(t) - d(t) - w(t)) - (\psi_d + \psi_i + \mu)v(t), \\ &\psi_i v(t) - (\gamma + \mu)w(t), \\ &\eta(1 - v(t) - d(t) - w(t))w(t) + \gamma w(t) + \psi_d v(t) - \mu d(t)]^T. \end{aligned}$$

Note that Ω is obviously a compact set. We only need to prove that if $y(0) \in \Omega$, then $y(t) \in \Omega$ for all $t \geq 0$. Note that $\partial\Omega$ consists of three plane segments:

$$\begin{aligned} P_1 &= (v, w, 0)|u, w \in [0, 1], v + w \leq 1, \\ P_2 &= (v, 0, d)|v, d \in [0, 1], v + d \leq 1, \\ P_3 &= (0, w, d)|w, d \in [0, 1], w + d \leq 1, \\ P_4 &= (v, w, d) \in R^3_+|v + w + d = 1, \end{aligned}$$

which have $v_1 = (0, 0, -1), v_2 = (0, -1, 0), v_3 = (-1, 0, 0), v_4 = (1, 1, 1)$ as their outer normal vectors, respectively. If the dot product of $f(y)$ and normal vectors (v_1, v_2, v_3, v_4) of the boundary lines are less than or equal to zero, then $y(t) \in \Omega$ for all $t \geq 0$. So,

$$\begin{aligned} &\langle f(y(t))|_{y\in p_1}, v_1\rangle = -\eta[1 - v(t) - w(t)]w(t) - \gamma w(t) - \psi_d w(t) + \mu d(t) \leq 0, \\ &\langle f(y(t))|_{y\in p_2}, v_2\rangle = -\psi_i v(t) \leq 0, \\ &\langle f(y(t))|_{y\in p_3}, v_3\rangle = -\beta w(t)[1 - d(t) - w(t)] \leq 0, \\ &\langle f(y(t))|_{y\in p_4}, v_4\rangle = 0. \end{aligned}$$

The proof is complete. □

Hence, system (1) is considered mathematically and physically well posed in Ω [21].

3 Theoretical analysis of the model

According to the model (2), we can calculate racism free and racism prevalence equilibrium. In the absence of racism, the racism free equilibrium point E_0 is calculated, which is $(u^*, v^*, w^*, d^*) = (1, 0, 0, 0)$. The existence of racism prevalence equilibrium point is computed after we have the threshold value R_0.

3.1 Basic reproduction number

Let $X = (v, w, u, d)^T$. System (2) can be written as [19]

$$\frac{dX}{dt} = \mathcal{F}(X) - \mathcal{V}(X),$$

where

$$\mathcal{F}(X) = \begin{pmatrix} \beta u(t)w(t) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \mathcal{V}(X) = \begin{pmatrix} (\psi_d + \psi_i + \mu)v(t) \\ -\psi_i v(t) + (\gamma + \mu)w(t) \\ \mu - (\beta + \eta)u(t)w(t) + \mu u(t) \\ \mu d(t) - [\eta u(t)w(t) + \gamma w(t) + \psi_d v(t)] \end{pmatrix}.$$

The Jacobian matrices of $\mathcal{F}(X)$ and $\mathcal{V}(X)$ at the racism free equilibrium E_0 are, respectively,

$$D\mathcal{F}(E_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}, \ D\mathcal{V}(E_0) = \begin{pmatrix} V & 0 \\ J_1 & J_2 \end{pmatrix},$$

where

$$F = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \psi_i + \psi_d + \mu & 0 \\ -\psi_i & \gamma + \mu \end{pmatrix}.$$

Also, $FV^{-1} = \begin{pmatrix} \frac{\psi_i \beta}{(\psi_i + \psi_d + \mu)(\gamma + \mu)} & \frac{\beta}{(\gamma + \mu)} \\ 0 & 0 \end{pmatrix}$ is the next generation matrix of system (2). Thus, the spectral radius of the matrix FV^{-1} can be computed as

$$\rho(FV^{-1}) = \frac{\psi_i \beta}{(\psi_i + \psi_d + \mu)(\gamma + \mu)}.$$

Hence, the basic reproduction number of system (2) is given by

$$R_0 = \frac{\psi_i \beta}{(\psi_i + \psi_d + \mu)(\gamma + \mu)}.$$

Now we can find the existence of racism prevalence equilibrium point E^*, when $w^* \neq 0$, which is

$$E^* = (u^*, v^*, w^*) = \left(\frac{1}{R_0}, \frac{b\mu(R_0 - 1)}{\psi_i(\beta + \eta)}, \frac{\mu(R_0 - 1)}{\beta + \eta} \right).$$

3.2 Stability analysis of the equilibrium

In the absence of the racism, the model has a unique racism free equilibrium E_0. To show the local stability of an equilibrium point E_0, we use the linearization principle. Linearity of the model system is calculated through Jacobin matrix. Therefore the system (2) is given by

$$J(u,v,w)=\begin{bmatrix}-(\beta+\eta)w(t)-\mu & 0 & -(\beta+\eta)u(t)\\ \beta w(t) & -(\psi_i+\psi_d+\mu) & \beta u(t)\\ 0 & \psi_i & -(\gamma+\mu)\end{bmatrix}.$$

Theorem 1. *If $R_0<1$, then the racism-free equilibrium E_0 of system (2) is locally asymptotically stable, and it is unstable when $R_0>1$.*

Proof. The Jacobin matrix of the model at E_0 is

$$J(1,0,0)=\begin{bmatrix}-\mu & 0 & -(\beta+\eta)\\ 0 & -(\psi_i+\psi_d+\mu) & \beta\\ 0 & \psi_i & -(\gamma+\mu)\end{bmatrix}. \tag{4}$$

Now we need to find the eigenvalues of the system from the Jacobian matrix (4). We obtain the characteristic polynomial

$$P(\lambda)=(-\mu-\lambda)^2\left(\lambda^2+(a+b)\lambda+ab(1-R_0)\right) \tag{5}$$

where $a=\psi_i+\psi_d+\mu$ and $b=\gamma+\mu$.

Obviously from equation $(-\mu-\lambda)^2$, we obtain $\lambda_{1,2}=-\mu$, which is real negative. We get the remaining real negative eigenvalues from $\lambda^2+(a+b)\lambda+ab(1-R_0)$, when $R_0<1$. Thus, E_0 is locally asymptotically stable where $R_0<1$. If $R_0>1$ some eigenvalues are non-negative, so E_0 is unstable. The proof is completed. □

Theorem 2. *If $R_0\leq 1$, then the racism-free equilibrium E_0 of system (2) is globally stable.*

Proof. Let $E^*=(v^*,w^*,d^*)$, where $v^*,w^*,d^*\in(0,1)$ be the equilibrium point of the system (3). Then, to obtain its explicit form, we need to solve the following system

$$\begin{aligned}&\beta w^*\left(1-v^*-d^*-w^*\right)-(\psi_d+\psi_i+\mu)v^*=0,\\ &\psi_i v^*-(\gamma+\mu)w^*=0,\\ &\eta(1-v^*-d^*-w^*w^*+\gamma w^*+\psi_d v^*-\mu d^*=0.\end{aligned} \tag{6}$$

From the first equation of system (6), we obtain

$$v^* = \frac{\beta w^*(1 - v^* - d^* - w^*)}{\psi_d + \psi_i + \mu}. \tag{7}$$

By substituting Eq.(7) into the second equation of (6), we can obtain the following equation

$$\left[\psi_i \frac{\beta(1 - v^* - d^* - w^*)}{\psi_d + \psi_i + \mu} - (\gamma + \mu)\right] w^* = 0. \tag{8}$$

From the Eq.(8), we can obtain that $w^* = 0$, or

$$w^* = \frac{R_0 - 1}{R_0} - (v^* + d^*). \tag{9}$$

The case $w^* = 0$ easily gives to the unique racism-free equilibrium $E_0 = (0, 0, 0)$ of system(3), it is consistent with Theorem 1. For the other case, if $R_0 \leq 1$, then w^* is obviously non-positive. This implies that there is no racism-prevalence equilibrium point. Hence, the racism-free equilibrium E_0 of the model is globally stable. The proof is completed. □

Theorem 3. *If $R_0 > 1$, then the racism prevalence equilibrium point E^* of system (2) is locally asymptotically stable.*

Proof. The Jacobian matrix of the model at E^* is

$$J\left(\frac{1}{R_0}, \frac{b\mu(R_0 - 1)}{\psi_i(\beta + \eta)}, \frac{\mu(R_0 - 1)}{\beta + \eta}\right) = \begin{bmatrix} -\mu R_0 & 0 & \frac{-(\beta+\eta)}{R_0} \\ \frac{\beta b\mu(R_0 - 1)}{\psi_i(\beta + \eta)} & -a & \frac{\beta}{R_0} \\ 0 & \psi_i & -b \end{bmatrix}. \tag{10}$$

Now we need to find the eigenvalues of the system from the Jacobian matrix (10). We obtain the characteristic polynomial

$$P(\lambda) = \lambda^3 + (a + b + \mu R_0)\lambda^2 + (a + b)\mu R_0 \lambda + \frac{\beta\mu b(R_0 - 1)}{R_0}. \tag{11}$$

By Routh-Hurwitz criteria, all eigenvalues are real and negative [7], where $R_0 > 1$. Therefore the system (2) is locally asymptotically stable at the racism prevalence equilibrium point E^*. The proof is complete. □

4 Numerical solution and analysis

In this section, we analyze numerically the dynamics of model (2). Specifically, some numerical examples are designed to illustrate the theoretical model and results of the previous sections. Note that the parameter values in the following simulations are not from real data but are just chosen to illustrate the above results. To make the problem physically interesting, the initial values of system (2) are set to be positive, i.e., $u(0) > 0, v(0) > 0, w(0) > 0$ and $d(0) > 0$.

Example 1. *Consider system (2) with initial values* $(u(t), v(t), w(t), d(t)) = (0.9, 0.06, 0.04, 0)$ *and* $\mu = 0.01$. *Then, it ensures the following two conditions:*

(a) If $R_0 = 0.54$, *by considering the parameter values* $\beta = 0.3, \eta = 0.2, \psi_i = 0.5, \psi_d = 0.4, \gamma = 0.3$ *and* $\mu = 0.01$, *Theorems (1,2) ensures that the racism free equilibrium point* E_0 *is asymptotically stable.*

(b) If $R_0 = 15$, *by considering the parameter values* $\beta = 0.3, \eta = 0.2, \psi_i = 0.5, \psi_d = 0.4, \gamma = 0.001$ *and* $\mu = 0.01$, *Theorem 3 confirm that the racism prevalence equilibrium point* E^* *locally asymptotically stable.*

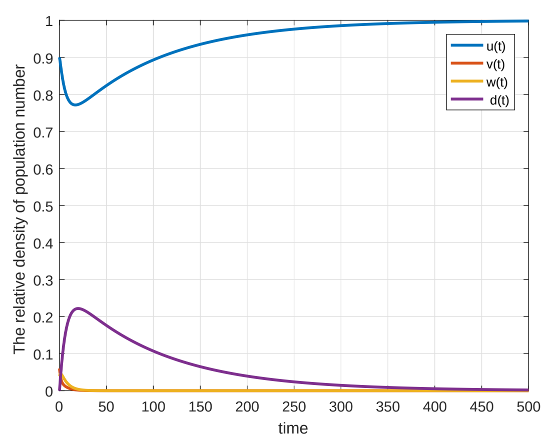

Figure 2: Some solutions of system (2) with parameters given in Example 1(a).

For system (2) with the initial values given in Example 1, Figure 2 shows the evolutions of $u(t), v(t), w(t)$ and $d(t)$ with the parameter value given in condition (a) of Example 1. Here we can observe that the relative density of $I, E, \& D$ nodes asymptotically converges to null, while the S-node asymptotically converge to unity. This implies that the racism spreading would disappear, which is supported by Theorem 1.

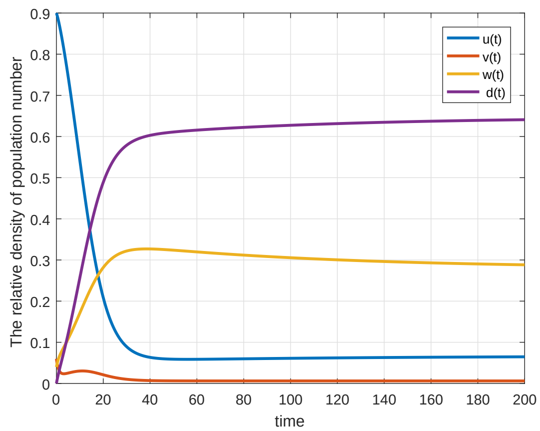

Figure 3: Some solutions of system (2) with parameters given in Example 1(b).

Figure 3 shows the simulations of $u(t), v(t), w(t)$ and $d(t)$ which tends to the constants $0.0647, 0.0062, 0.2883$ and 0.6408, respectively. This implies that in this case cyber racism will finally keep propagating through the network at a steady level.

In order to understand the behavior of racism spreading dynamics, we need to deeply analyze the parameters of the model (2), and further explore how these parameters exactly influence the spreading processes. Clearly, the parameter β describes the probability that S-node converts to be an E-node by a single I-node per unit time. It is essentially determined by the rate of an I-node spreading the racist ideology. The following example is designed to numerically illustrate the results of $w(t)$ with several different values of β.

Example 2. *Consider system (2) with parameters given by* $\eta = 0.1, \psi_i = \psi_d = 0.5, \gamma = 0.01$ *and* $\mu = 0.01$. *Figure 4 numerically illustrates the evolutions of the percentage of* I*-nodes with different* β.

In Figure 4 the prevalence of $w(t)$ versus t corresponding to different infected rates β, which are chosen as $\beta = 0.01, \beta = 0.1, \beta = 0.4, \beta = 0.9$ from bottom to top, can be observed. This simulation finally converges to corresponding constants, respectively. More specifically, we can see that $w(t)$ increases significantly as β increases. This describes that when β values reduce, similarly the spreaders or racist nodes also reduce. Thus,

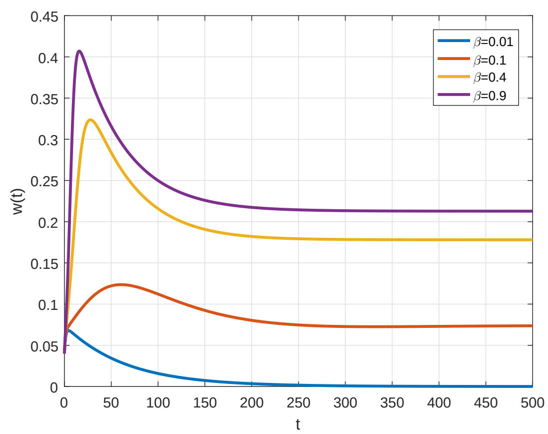

Figure 4: Evolutions of $w(t)$ in system (2) with varying over different infected rate β and other parameters specified in Example 2. Colors represent different values of β.

the reduction of the contact rate β is helpful for the controlling of cyber racism propagation.

The parameter ψ_i is introduced to depict the dynamic transitions between the I-nodes and E-nodes. That is, ψ_i represents the probability of an E-node turning into an I-node. The influence of the parameter ψ_i is observed in Example 3.

Example 3. *Consider system (2) with parameters given by* $\beta = 0.3, \eta = 0.1, \psi_d = 0.3, \gamma = 0.01$ *and* $\mu = 0.01$*. Figure 5 numerically illustrates the evolutions of the percentage of I-nodes with different* ψ_i*.*

Figure 5 shows that the impacts of the ψ_i, which are chosen as $\psi_i = 0.01, \psi_i = 0.1, \psi_i = 0.2, \psi_i = 0.5$ from bottom to top, on the density of I-nodes. We can see that the density of I-nodes increases firstly, reaches the peak, then descends, finally tends to the steady state. This implies that, when the transition rate from E-nodes to I-nodes is approaches to unity, similarly the racist density becomes to large. The reduction of ψ_i is support to control the racism propagation.

Additional ψ_d is also the parameter that represent the probability of transition from I-nodes to D-nodes. The effects of ψ_d is illustrated by Example 4.

Example 4. *Consider system (2) with parameters specified by* $\beta = 0.3, \eta = 0.1, \psi_i = 0.3, \gamma = 0.01$ *and* $\mu = 0.01$*. Figure 6 numerically illustrates the evolutions of* $w(t)$ *with several values of* ψ_d*.*

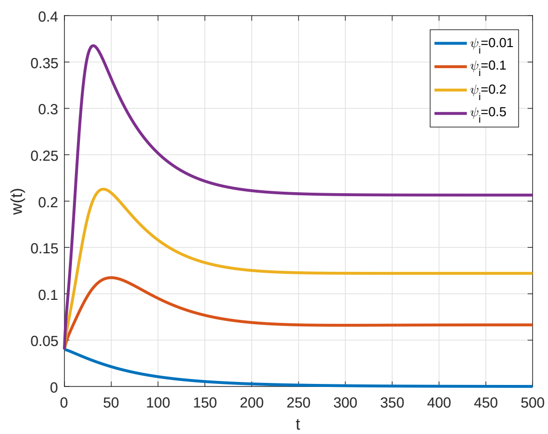

Figure 5: Evolutions of $w(t)$ in system (2) with varying ψ_i and other parameters specified in Example 3. Colors represent different values of ψ_i.

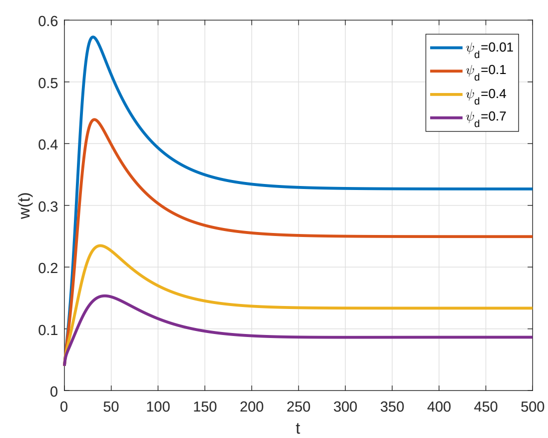

Figure 6: Evolutions of $w(t)$ in system (2) with varying ψ_d and other parameters specified in Example 4. Colors represent different values of ψ_d.

Figure 6 illustrates how the density of I-nodes changes over time with different transmission probability ψ_d from racist nodes to the racism-denier nodes, such as $\psi_d = 0.01, \psi_d = 0.1, \psi_d = 0.4, \psi_d = 0.7$. In this figure we can see that I-nodes can quickly reach to the peak in the shorter time with the lower transmissions rate. This implies that rising values of ψ_d can reduce the racist node density and helps to control the propagation of racism.

Once racism-believed users realize that racism is not good, they will no longer believe in it again. Thus, the parameter γ is introduced to depict this situation that a racism-believed user turns to be a racism-denied user at a certain probability.

Example 5. *Consider system (2) with parameters specified by* $\beta = 0.3, \eta = 0.1, \psi_i = 0.4, \psi_d = 0.2$ *and* $\mu = 0.01$. *We illustrate the significant effect of* γ *on the propagation of racism.*

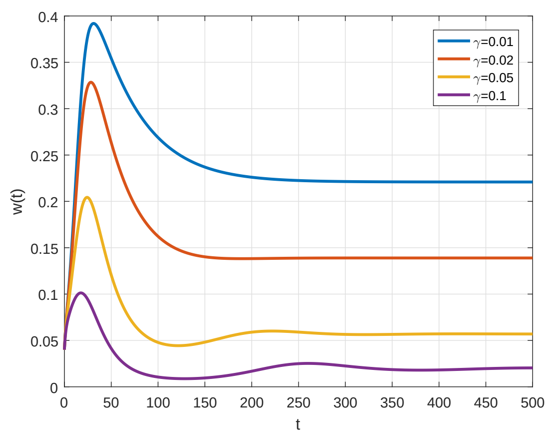

Figure 7: Evolutions of $w(t)$ in system (2) with varying γ and other parameters specified in Example 5. Colors represent different values of γ.

Figure 7 shows the prevalence of $w(t)$ vs time corresponding to different γ values, which are chosen as $\gamma = 0.01, \gamma = 0.0.02, \gamma = 0.0.05, \gamma = 0.1$ from top to bottom. We can also observe that the value of $w(t)$ will finally keep steady at a lower value for a greater value of γ. This indicates that the parameter γ has a significant impact on the evolutionary dynamics of the model (2). For the case in Example 5, the simulations of Figure 7 suggests that it would be much better to control the value of parameter $\gamma > 0.1$ so as to keep the final percentage of I-nodes at a slight level.

All racism susceptible nodes do not accept racism when they receive the message from I-nodes. These users reject racism and they will come to D-nodes. This, activities are represented by the parameter η.

Example 6. *Consider system (2) with parameters specified by* $\beta = 0.3, \psi_i = 0.4, \psi_d = 0.3, \gamma = 0.01$ *and* $\mu = 0.01$. *We illustrate the significant effect of* η *on the propagation of racism.*

In Figure 8 the prevalence $w(t)$ versus t corresponding to different deny rates η, which are chosen as $\eta = 0.01, \eta = 0.1, \eta = 0.4, \eta = 0.9$ from bottom

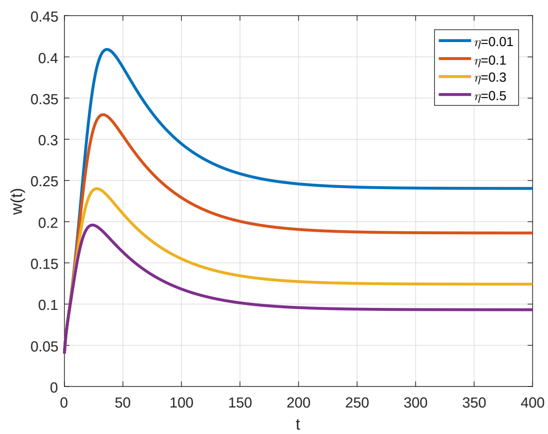

Figure 8: Evolutions of $w(t)$ in system (2) with varying η and other parameters specified in Example 6. Colors represent different values of η.

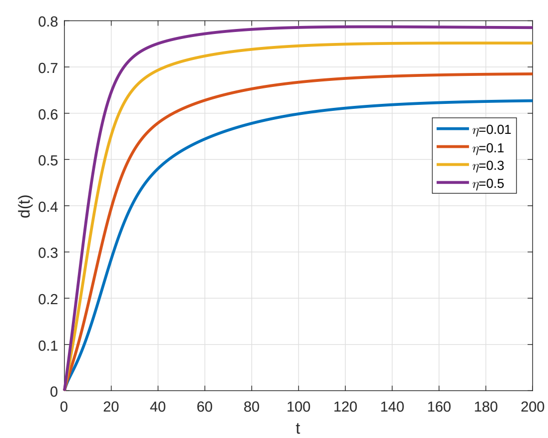

Figure 9: Evolutions of $d(t)$ in system (2) with varying η and other parameters specified in Example 6. Colors represent different values of η.

to top, is illustrated. We can easily find that the density of the I-nodes decrease as η increases. While, in Figure 9 we illustrate the prevalence $d(t)$ with the same choice as Figure 8. It shows that the D-nodes increase as η values increase. This indicates that when we increase the value of η, the racism propagation will reduce.

All the above numerical simulations illustrate the correctness of the theoretical results. Through these simulation results, we can conclude that the spreading of cyber racism can be effectively inhibited by making suitable measures to control the model parameters.

5 Conclusions

This paper is mainly focused on the dynamics of racism spreading on social networks. Nowadays racism is a global issue, so to study the spread dynamics has a mimic role to set up a control measure of it. This paper is a first article which is a study about cyber racism in the mathematical modeling context. Mainly we follow the compartmental approach to study the propagation process.

A new $SEID$ racism spreading model has been formulated and analyzed theoretically and numerically. By using the next-generation matrix approach, we obtained the basic reproduction number of R_0. As the results indicate, the basic reproduction number in social networks is virtually correlated to the propagation of racism. As a result of theoretical analysis and the supporting illustration through the numerical solution in Example 1 of Figure 2 when $R_0 \leq 1$ the racism propagation is die-out (converges to zero). While, Figure 3 reflect when $R_0 > 1$ racism prevalence equilibrium point is stable and racism is spread over the cyberspace.

The sensitivity analysis of the parametric values is performed. Reducing β, ψ_i values are useful to eradicate the racism spread. While, rising of γ, η, ψ_d values have a significant impact to control the racism spread.

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