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Global dynamics of a mathematical model on smoking: impact of anti-smoking campaign

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Abstract. We propose and analyze a mathematical model to study the dynamics of smoking behavior under the influence of educational and media programs. Proposed mathematical model subdivides the total population into potential smokers, smokers and those smokers who quit smoking permanently. The biologically feasible equilibrium points are computed and their stability is analyzed and discussed. The theoretical analysis of the model reveals that the smoking-free equilibrium is stable when a threshold, termed as the smokers-generation number, is less than unity, and unstable if this threshold value is greater than unity. Moreover, number of smokers may be effectively controlled by keeping the smokers generation number less than unity. Analytical findings are justified by numerical simulation.

Keywords: Smoking, education, media, global Stability, Lyapunov function.
AMS Subject Classification: 34D23, 34D20, 93D30.

1 Introduction

Smoking is a leading cause of heart disease, strokes, peripheral vascular diseases, chronic obstructive lung diseases and other respiratory diseases. Moreover, it is also a probable cause of peptic ulcer diseases and increased infant mortality including sudden infant death syndrome (SIDS). Every

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year, smoking causes some 5 million premature deaths[1] . Smoking in India has been known since 2000 BC and is first mentioned in the Atharvaveda, which dates back a few hundred years BC. It reveals that smoking has been practiced in India for at least 2,000 years. Tobacco was introduced to India in the 17th century that later on merged with other existing practices of smoking [11].As a consequence of after impact of smoking, government took many steps to control smoking. Government of India has launched many antismoking campaigns in the country. Law is enforced to stop individuals smoking in indoor public places, such as bars, pubs and restaurants. Smoking was prohibited in public places nationwide from 2 October 2008 [11]. Kerala became the first Indian state to ban smoking in public places on 12 July 1999 after the declaration of a Division Bench of the Kerala High Court. Since 8 September 2000, The Cable Television Network (Regulation) Amendment Bill completely prohibits cigarette and alcohol advertisements[2]. The government began screening two anti-tobacco advertisements with effect from 2 October 2012, titled "Sponge" and "Mukesh", in movie theatres and on television too[3]. It is also made mandatory for theatres to display a disclaimer on-screen whenever smoking scenes are showed in the movie [32]. Later on, the "Sponge" and "Mukesh" ads were replaced by "Child" and "Dhuan"[4]. Although mathematical modeling has been used extensively to address questions of public health importance in the pioneering works of Bernoulli, Kermack and McKendrick [4,23–25] and those reported in literature [1–3,6–10,14–16,19–22,28–31,33–36] not much work has been done in terms of the mathematical modeling of human social behavior. In particular, except the basic model in [5,6,13,18,26,37–39], no mathematical study has been found that fully examine, and assesses, the impact of smoking in population. However, Castillo-Garsow et al. [11] proposed a simple mathematical model on smoking. They considered a system of ordinary differential equations described by the simplified PSQ model. In this mathematical model, total constant population is divided into three classes, potential smoker, smokers and smoker who have quit smoking permanently. Later, this mathematical model was modified by Sharomi and Gumel (2008) [33], by introducing a new class of smokers who temporarily quit smoking. In 2012, Erturk et al. [17] studied this model numerically after introducing fractional derivatives into it. The aim of this paper is

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[1] World Health Organization, http://www.emro.who.int/tfi/facts.htm, fact2, 2010.

[2] Alcohol in India, http://www.ias.org.uk, 8 September 2000. Retrieved 14 January 2013.

[3] Government to launch fresh campaign against smoking, https://economictimes.indiatimes.com, 2013.

[4] New ad spots to focus on passive smoking, http://shodhganga.inflibnet.ac.in/bitstream/10603/97366/7/references.pdf, 2013.

to provide a qualitative study of the dynamics of smoking with impact of anti-smoking campaigns. Inclusion of anti-smoking campaigns is the novel feature of our model that has not been found in the literature to the best of our knowledge. In this paper, we study the dynamics of progression of potential smoker class to the class of smokers followed by the movement of smokers to the state in which they permanently quit smoking due to anti-smoking campaigns.

2 Mathematical Model

We propose a mathematical model to asses the impact of anti-smoking campaigns to control smoking. We considered a region with total population T at any time t. The whole population is divided in three subclasses: Potential smokers (P), Smokers (S), and Permanent smoking quitters (R), i.e. those individuals who have either quit smoking or never smoke due to anti-smoking campaigns. If the individuals are well-informed about the fatality caused by smoking, then they may be refrained from smoking. Here we assume that due to awareness individuals in potential smokers class will leave at a rate δp. These individuals will never smoke and hence will join permanent quitters class. Let μ be inflow rate of individuals in potential smoker class. It also represent natural per capita death rate in each compartment. New smokers increase through contact of potential smokers with smokers, which is taken as standard mass action inside. Let β be the rate of transmission of smoking habit, so that βPS denotes the smoking incidence rate. A fraction $\alpha(1-\epsilon)S$ of these recovered will revert back to potential smoker class due to low determination level and remaining $\alpha\epsilon S$ will proceed to removed class. The ϵ is the measure of determination. On the other hand, some of the smokers become aware of ill-effects of smoking due to media campaigns, and join the quitters class at a rate ηSM. Further, it assumed that some media campaigns fade or loss of their impact on people, moreover ϕ and ϕ_0 are constants denoting the rate of implementation and fading of media campaigns respectively, but a baseline number of media campaigns M_0 should be maintained in the system.

$$\frac{dP}{dt} = \mu - \beta PS + \alpha(1-\epsilon)S - \mu P - \delta P, \tag{1}$$

$$\frac{dS}{dt} = \beta PS - \mu S - \alpha S - \eta SM, \tag{2}$$

$$\frac{dR}{dt} = \alpha\epsilon S + \eta SM - \mu R + \delta P, \tag{3}$$

$$\frac{dM}{dt} = \phi S - \phi_0(M - M_0), \tag{4}$$

where $P(0) \geq 0$, $S(0) \geq 0$, $R(0) \geq 0$ and $M(0) \geq 0$. Here note that

$$P(t) + S(t) + R(t) = T(t).$$

By combining the first three equations of the above model, we have

$$\frac{dT}{dt} = \mu - \mu T,$$

which implies that the total population $\lim_{t\to\infty} \sup T(t) \leq 1$. Since the above model monitors human population, all the variables and parameters are assumed to be non-negative for all $t \geq 0$. We study the above model in the positively invariant set

$$\Omega = \{(P, S, R, M) \in R^4_+ : 0 \leq P, S, R \leq 1, 0 \leq M \leq \frac{\phi}{\phi_0}\},$$

which is region of attraction for model.

3 Equilibria and their stability analysis

3.1 Equilibria

3.1.1 Smoking-free equilibrium point (SFE)

The Smoking-Free equilibrium point (SFE) of the system (1-4), is obtained by setting all the smokers classes and recovered classes to zero. We get

$$\begin{aligned} \mu - \mu P^0 - \delta P^0 &= 0, \\ \mu R^0 - \delta P^0 &= 0, \\ \phi_0(M - M_0) &= 0, \end{aligned}$$

which yields $P^0 = \mu/(\mu + \delta)$, $R^0 = \delta/(\mu + \delta)$, $M^0 = M_0$. The SFE point for our system is given by

$$E_1 = (P^0, 0, R^0, M^0) = \left(\frac{\mu}{(\mu + \delta)}, 0, \frac{\delta}{(\mu + \delta)}, M_0\right).$$

Thus, system becomes smoking free when $M = M_0$, i.e., program execuation is 100% which is an ideal situation.

3.1.2 Existence of endemic equilibrium point

The model system exhibits an smoking present equilibrium $E_2(P^*, S^*, R^*, M^*)$. By setting $\frac{dP}{dt} = \frac{dS}{dt} = \frac{dR}{dt} = \frac{dM}{dt} = 0$, all components of E_2 can be expressed in terms of S^* as

$$M^* = M_0 + \frac{\phi}{\phi_0}S^*, \quad P^* = \frac{\mu + \alpha(1-\epsilon)S^*}{\mu + \delta + \beta S^*},$$
$$R^* = \frac{1}{\mu}\left(\alpha\epsilon S^* + \eta S^*\left(M_0 + \frac{\phi}{\phi_0}S^*\right) + \delta\left(\frac{\mu + \alpha(1-\epsilon)S^*}{\mu + \delta + \beta S^*}\right)\right)$$

and $P^* = (\mu + \alpha + \eta M^*)/\beta$. So from the above relation we can write

$$\frac{\mu + \alpha(1-\epsilon)S^*}{\mu + \delta + \beta S^*} = \frac{\mu + \alpha + \eta M^*}{\beta}.$$

Now putting the value of M^* in this equation and simplifying gives a quadratic equation with respect to S^* as $A_1 S^{*2} + A_2 S^* - A_3 = 0$, where

$$A_1 = \beta\eta(\frac{\phi}{\phi_0}),$$
$$A_2 = \left(\mu\beta + \alpha\beta\epsilon + \beta\eta M_0 + (\mu + \delta)\eta(\frac{\phi}{\phi_0})\right),$$
$$A_3 = (\mu + \delta)(\eta M_0 + \mu + \alpha)[S_0 - 1].$$

It is clear that sign of A_1 and A_2 is always positive and sign of A_3 depends on $(S_0 - 1)$, where

$$S_0 = \frac{\mu\beta}{(\mu + \delta)(\eta M_0 + \mu + \alpha)}.$$

Let $S^*_\pm$ denote the roots of the above quadratic equation, then

$$S^*_\pm = \frac{-A_2 \pm \sqrt{A_2^2 + 4A_1A_3}}{2A_1}.$$

We now consider the following three cases:

Case 1: If $S_0 > 1$, then $A_3 > 0$. In this case S^*_- is always negative and S^*_+ is always positive. It follows that above quadratic equation has a unique positive solution and there exists a unique positive equilibrium E_2 whenever $S_0 > 1$.

Case 2: If $S_0 < 1$, then $A_3 < 0$. Here both the roots of quadratic equation S^*_+ and S^*_- positive if $A_2 < 0$ and $A_2^2 + 4A_1A_3 > 0$; otherwise there is not any positive roots of the equation exist. This implies that, multiple endemic equilibia (E^*_+ and E^*_-) exist when $S_0 < 1$.

Case 3: If $S_0 = 1$, then $A_3 = 0$ and $A_1 > 0$, $A_2 > 0$. Thus there are no positive solutions.

3.2 Smokers's generation number

The quantity S_0 plays the role analogous to the basic reproduction number in epidemic models. In the mathematical epidemiology, the basic reproduction number S_0 defined as the expected number of secondary infections arising from a single individual during his or her entire infectious period, in a population of susceptible [14]. A fundamental result in mathematical epidemiology, which most epidemic model follow, is that endemic equilibrium exists whenever $S_0 > 1$, which physically means that the disease can invade the population if each infective generates, on average, more than one new infective cases in the population. Following similar pattern, smokers generation number S_0 is determined as

$$S_0 = \frac{\mu\beta}{(\mu+\delta)(\eta M_0+\mu+\alpha)},$$

which demonstrates that one smoker creates $\mu\beta$ smokers in its whole life time of $1/(\eta M_0+\mu+\alpha)$ spent in smoking class. Life time spent in smoking class can be reduced by increasing ηM_0 and hence S_0 can be reduced by increase ηM_0. Thus anti-smoking campaign may play an important role in reducing number of smokers.

3.3 Stability Analysis

Now we proceed to study the stability behavior of equilibria E_1 and E_2. The Jacobian matrix at SFE is given by

$$J_1^* = \begin{bmatrix} -\mu-\delta-\lambda & -\beta P+\alpha(1-\epsilon) & 0 & 0 \\ 0 & \beta P-\mu-\alpha-\eta M_0-\lambda & 0 & 0 \\ \delta & \alpha\epsilon+\eta M_0 & -\mu-\lambda & 0 \\ 0 & \phi & 0 & -\phi_0-\lambda \end{bmatrix}. \quad (5)$$

It is apparent that eigenvalues of J_1^* is $(-\mu-\delta)$, $-\phi_0$, $-\mu$ and $\beta P-\mu-\alpha-\eta M_0$. It is noted that if

$$S_0 = \frac{\mu\beta}{(\mu+\delta)(\eta M_0+\mu+\alpha)} < 1,$$

then all the eigenvalues of J_1^* are negative whereas one eigenvalue becomes positive if $S_0 > 1$. Hence we make an assertion that Smoking-free equilibrium point is locally asymptotically stable if $S_0 < 1$ and unstable (saddle point) if $S_0 > 1$. Therefore the Smoking-free equilibrium point undergoes a trancritical bifurcation if $S_0 = 1$.

Now for the endemic equilibrium the Jacobian matrix is given as

$$J_2^* = \begin{bmatrix} -\mu-\beta S^*-\delta-\lambda & -\beta P^*+\alpha(1-\epsilon) & 0 & 0 \\ \beta S^* & \beta P^*-\mu-\alpha-\eta M^*-\lambda & 0 & -\eta S^* \\ \delta & \alpha\epsilon+\eta M^* & -\mu-\lambda & \eta S^* \\ 0 & \phi & 0 & -\phi_0-\lambda \end{bmatrix}. \quad (6)$$

By expanding along C_3 we get

$$(-\mu-\lambda)\begin{bmatrix} -\mu-\beta S^*-\delta-\lambda & -\beta P^*+\alpha(1-\epsilon) & 0 \\ \beta S^* & \beta P^*-\mu-\alpha-\eta M^*-\lambda & -\eta S^* \\ 0 & \phi & -\phi_0-\lambda \end{bmatrix} = 0. \quad (7)$$

Clearly one eigenvalue is $-\mu$ of J_2^* and other three eigenvalues are found by solving the characteristic polynomial for J_2^* is given by

$$\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0,$$

where

$$\begin{aligned} B_1 &= \phi_0 + (\beta P^* - \mu - \alpha - \eta M^*) + (\beta S^* + \mu + \delta) \\ B_2 &= \beta S^*(\beta P^* - \alpha(1-\epsilon)) + \eta S^*\phi - (\beta P^* - \mu - \alpha - \eta M^*)\phi_0 \\ &\quad +(\beta S^* + \mu + \delta)\phi_0 - (\beta S^* + \mu + \delta(\beta P^* - \mu - \alpha - \eta M^*)), \\ B_3 &= \beta S^*(\mu + \alpha + \eta M^*)\phi_0 + (\mu+\delta)(\mu+\alpha+\eta M^*)\phi_0 \\ &\quad -(\mu+\delta)\beta P^*\phi_0 - \phi_0\beta S^*\alpha(1-\epsilon) + \beta\eta S^{*2}\phi + (\mu+\delta)\eta S^*\phi. \end{aligned}$$

By using the Routh-Hurwitz Criterion Marsden and McCracken (1976) [27], we can say that endemic equilibrium point E_2 is locally asymptotically stable if and only if the inequalities, $B_i > 0, i = 1, 2, 3$ and $B_1B_2 - B_3 > 0$ are satisfied.Thus on the basis of study regarding the equilibria of given model and their stability following results have obtained:

Theorem 1. (i) *If $S_0 < 1$, then (SFE) E_1 is locally asymptotically stable.* (ii) *If $S_0 > 1$, then Smoking-free equilibrium point (SFE) E_1 becomes unstable and the endemic equilibrium E_2 exists, which is locally asymptotically stable if the inequality $B_1B_2 - B_3 > 0$ holds.*

4 Global stability

Consider the following Lyapunov's function as

$$V = \frac{1}{2}(P-P^*)^2 + \frac{q_1}{2}(S-S^*)^2 + \frac{q_2}{2}(R-R^*)^2 + \frac{q_3}{2}(M-M^*)^2,$$

where q_1, q_2 and q_3 are some positive constants to be chosen later. On differentiating V with respect to t along the solutions of given model, we get

$$\begin{aligned}\frac{dV}{dt} &= -\frac{1}{2}b_{11}(P-P^*)^2 + b_{12}(P-P^*)(S-S^*) - \frac{1}{3}b_{22}(S-S^*)^2 \\ &-\frac{1}{3}b_{22}(S-S^*)^2 + b_{24}(S-S^*)(M-M^*) - \frac{1}{2}b_{44}(M-M^*)^2 \\ &-\frac{1}{2}b_{44}(M-M^*)^2 + b_{34}(M-M^*)(R-R^*) - \frac{1}{3}b_{33}(R-R^*)^2 \\ &-\frac{1}{3}b_{33}(R-R^*)^2 + b_{23}(S-S^*)(R-R^*) - \frac{1}{3}b_{22}(S-S^*)^2 \\ &-\frac{1}{3}b_{33}(R-R^*)^2 + b_{13}(P-P^*)(R-R^*) - \frac{1}{2}b_{11}(P-P^*)^2.\end{aligned}$$

Now we choose $q_1 = q_2 = q_3 = 1$ and for the sufficient condition of $\frac{dV}{dt}$ to be negative definite, the following inequalities are hold:

$$\begin{aligned} b_{12}^2 &< \frac{2}{3}b_{11}b_{22}, \quad b_{24}^2 < \frac{2}{3}b_{22}b_{44}, \quad b_{34}^2 < \frac{2}{3}b_{33}b_{44}, \\ b_{23}^2 &< \frac{4}{9}b_{22}b_{33}, \quad b_{13}^2 < \frac{2}{3}b_{11}b_{33},\end{aligned}$$

where

$$\begin{aligned} b_{11} &= (\mu+\delta+\beta S^*), \quad b_{22} = q_1(\mu+\alpha+\eta M^* - \beta P), \quad b_{33} = q_2\mu, \\ b_{44} &= q_3\phi_0, \quad b_{12} = (\alpha(1-\epsilon) - \beta P + \beta S^* q_1), \quad b_{13} = q_2\delta, \\ b_{24} &= q_3\phi - q_1\eta S, \quad b_{23} = q_2(\alpha\epsilon + \eta M^*), \quad b_{34} = q_2\eta S.\end{aligned}$$

This shows that V is a Lyapunov's function for the model, implying that endemic equilibrium E_2 is globally asymptotically stable.

On the basis of our study regarding the endemic equilibrium E_2 of model and their global stability we have following result.

Theorem 2. *The model is globally or nonlinearly asymptotically stable around the endemic equilibrium point E_2, provided the following conditions hold:*
(i) $(\alpha(1-\epsilon) - \beta P + \beta S^* q_1)^2 < \frac{2}{3}(\mu+\delta+\beta S^*)q_1(\mu+\alpha+\eta M^* - \beta P)$,
(ii) $(q_3\phi - q_1\eta S)^2 < \frac{2}{3}q_1(\mu+\alpha+\eta M^* - \beta P)q_3\phi_0$,
(iii) $(q_2\eta S)^2 < \frac{2}{3}q_2\mu q_3\phi_0$,
(iv) $(q_2(\alpha\epsilon + \eta M^*))^2 < \frac{4}{9}q_1(\mu+\alpha+\eta M^* - \beta P)q_2\mu$,
(v) $(q_2\delta)^2 < \frac{2}{3}(\mu+\delta+\beta S^*)q_2\mu$,
inside the region of attraction Ω.

5 Numerical Simulations

In this section, we present computer simulation results for model by using MATLAB 7.10. We choose the following set of parameters:

$$\mu = 0.2, \beta = 0.9, \epsilon = 0.02, \delta = 0.1, \phi = 0.2, \eta = 0.03, \phi_0 = 0.01, M_0 = 3.$$

For these values of parameters, we see that the endemic equilibrium $E_2(P^*, S^*, R^*, M^*)$ exists and $E_2(P^*, S^*, R^*, M^*)$ are given as follows: $P^* = 0.5005, S^* = 0.1340, R^* = 0.3655$ and $M^* = 5.6806$.

We also note that all conditions of Theorems 1 and 2 are satisfied. This implies that E_2 is locally as well as globally asymptotically stable for the above set of values of parameters.

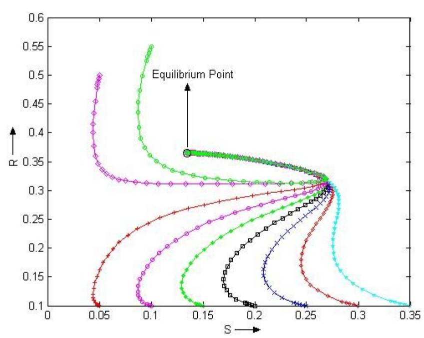

Figure 1: Global Stability in R-S Plane.

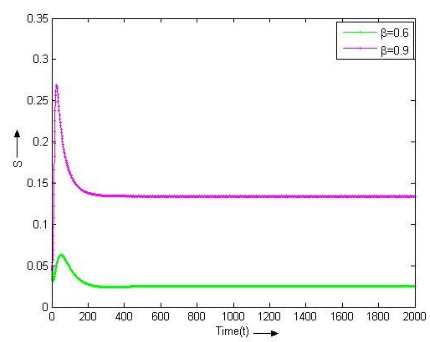

Figure 2: Variation of smokers population S with time for different values of parameter β.

In Figure 1, we have considered the five different initial values of the recovered and smokers populations. All trajectories starting from different initial values approach to (R^*, S^*). This point is independent of the initial status. This shows that (R^*, S^*) is globally asymptotically stable in RS-plane.

In Figure 2, we have shown the effect of rate of transmission β of smoking habit on smokers population we observe that smokers population increases with increase in rate of transmission of smoking habit.

In Figure 3, we have shown that education parameter δ increases , the number of smokers population decreases.

Figure 4, represent the effect of information dissemination rate η on smokers population. This shows that by increasing the dissemination rate of awareness programs, the number of smokers population decreases.

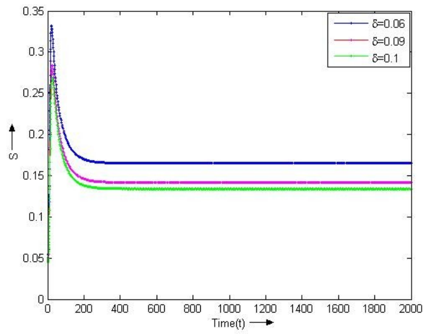

Figure 3: Variation of smokers population S with time for different values of parameter δ.

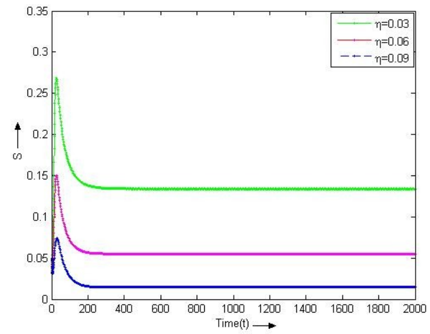

Figure 4: Variation of smokers population S with time for different values of parameter η.

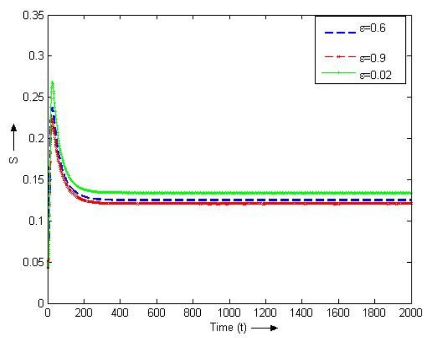

Figure 5: Variation of smokers population S with time for different values of parameter ϵ.

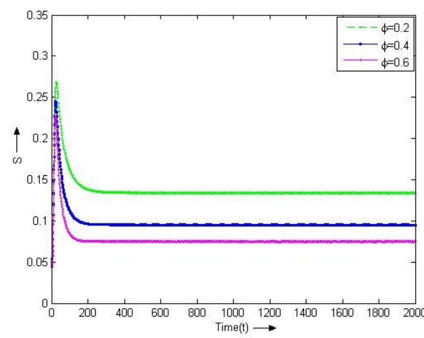

Figure 6: Variation of smokers population S with time for different values of parameter ϕ.

Figure 5, represent the effect of measure of determination ϵ increases, the number of smokers population decreases.

In Figure 6, we have shown that proportionality constant ϕ represent the rate at which the awareness campaigns by media are being implemented increases, the number of smokers population decreases.

6 Conclusion

In this paper, we have introduced a mathematical model to study the effect of education and awareness programs (run by media) on smokers population. The global dynamics of this model has been studied. We have shown that there exists only two equilibrium points; the smoking-free equilibrium (SFE) $E_1\left(\frac{\mu}{\mu+\delta}, 0, \frac{\delta}{\mu+\delta}, M_0\right)$, i.e. total elimination of smokers (as $S = 0$) and the smoking present equilibrium $E_2\left(P^*, S^*, R^*, M^*\right)$ i.e. smoking will persist. The (SFE) is locally asymptotically stable for reproductive number $S_0 < 1$ and the smoking present equilibrium exists for $S_0 > 1$ and is globally asymptotically stable under the conditions stated in Theorem 2. We have also carried out numerical simulations to validate the analytical results. We have shown that smokers population decreases as we increase the impact of education on potential smokers increases. When individuals are educated about the fatality of the diseases caused by smoking, they will refrain from smoking in future and hence will move to removed class. We note that of permanent quitters $S_0 = \frac{\mu\beta}{(\mu+\delta)(\eta M_0+\mu+\alpha)}$ in order to eradicate smoking we need to bring down S_0 below one. This is possible by increasing education and hence increasing education programs we can possibly eradicate the smoking. Also, it is recommended that continuous health education programs on smoking should be organized by institutions, associations and societies with in and outsides the school collegs and other public places as well because this will make them well informed towards hazards of smoking and induce in them. Hence anti smoking campaigns by education and media example play an important role to reduce the smoking.

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