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# Simulation of particle diffusion and heat transfer in a two-phase turbulent boundary layer using the Eulerian-Eulerian approach

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**Abstract.** This work investigates the response of two-dimensional, turbulent boundary layer characteristics over a flat plate to the presence of suspended particulate matter. Both phases are assumed to be interacting continua. That is, the carrier fluid equations are considered to be coupled with the particle-phase equations. A finite-difference technique with non-uniform grid has been employed for the solution of the governing equations and therefore, interpretation of the results and comparison of the present result with the results of other references. The results clearly demonstrate that the presence of particles damped the fluid turbulence and apparently affects the skin friction and heat transfer characteristics equally.

*Keywords:* Particulate suspensions, turbulent boundary layer characteristics, flat plate, finite difference techniques, shear stress, heat transfer.
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## Nomenclature

| | | |
|---|---|---|
| $(x, y)$ | $\rightarrow$ | Space coordinates i.e. distance along the perpendicular to plate length |
| $\vec{q}\ (u, v)$ | $\rightarrow$ | Velocity components for the fluid phase in x- and y- directions, respectively |
| $\vec{q_p}\ (u_p, v_p)$ | $\rightarrow$ | Velocity components for the particle phase in x- and y- directions, respectively |
| $(T, T_p)$ | $\rightarrow$ | Temperature of fluid and particle phase, respectively |
| $(T_w, T_\infty)$ | $\rightarrow$ | Temperature at the wall and free-stream, respectively |
| $(v, v_p)$ | $\rightarrow$ | Kinematic coefficient of viscosity of fluid and particle phase, respectively |
| $(\rho, \rho_p)$ | $\rightarrow$ | Density of fluid and particle phase, respectively |
| $(\rho_s, \rho_m)$ | $\rightarrow$ | Material density of particle and mixture, respectively |
| $(\mu_s, \mu_m)$ | $\rightarrow$ | Coefficient of viscosity of fluid and particle phase, respectively |
| $(\tau_p, \tau_T)$ | $\rightarrow$ | Velocity and thermal equilibrium time, respectively |
| $(c_p, c_s)$ | $\rightarrow$ | Specific heat of fluid and particle phase, respectively |
| $(\kappa, \kappa_s)$ | $\rightarrow$ | Thermal conductivity of fluid and particle phase, respectively |
| $(Re, Re_p)$ | $\rightarrow$ | Fluid and particle phase Reynolds number, respectively |
| $(Nu, Nu_p)$ | $\rightarrow$ | Fluid and particle phase Nusselt number, respectively |
| $Pr$ | $\rightarrow$ | Prandtl number |
| $Ec$ | $\rightarrow$ | Eckert number |
| $c_f$ | $\rightarrow$ | Skin friction coefficient |
| $\tau_w$ | $\rightarrow$ | Skin friction (Shear stress for clear fluid) |
| $\varphi$ | $\rightarrow$ | Volume fraction of Suspended particulate matter (SPM) |
| $P$ | $\rightarrow$ | Pressure of fluid phase |
| $D$ | $\rightarrow$ | Diameter of the particle |
| $\delta$ | $\rightarrow$ | Boundary layer thickness |
| $\alpha$ | $\rightarrow$ | Loading ratio |
| $\epsilon$ | $\rightarrow$ | Diffusion parameter |
| $F$ | $\rightarrow$ | Friction parameter between the fluid and the particle($F = \frac{18\mu}{\rho_p D^2}$) |
| $J_{max}$ | $\rightarrow$ | Maximum number of grid points along y-axis |
| $L$ | $\rightarrow$ | Reference Characteristic length |
| $W$ | $\rightarrow$ | Dummy variable |
| $r_y$ | $\rightarrow$ | Grid growth ratio |
| $r.m.s$ | $\rightarrow$ | Root mean square error |
| $U$ | $\rightarrow$ | Free stream velocity |
| $UB$ | $\rightarrow$ | Initial velocity profile of carrier fluid |

## 1 Introduction

Prediction of turbulence levels of a continuous phase in two-phase flows is important for modeling dispersion of solid particles or droplets, particle collisions or mixing of gaseous species. The gas-suspension flows have been obtained in Batchelor [1,2], Marble [15] and Soo [32] within the frame work of the Eulerian approach. The modeling of turbulent gas-suspension flows by using the Eulerian approach was obtained by many authors Gharraei et. al. [10], Han et al. [11], Hossain & Naser [13], Ozbelge & Eraslan [24] and Shotorban & Balachandar [31]. The effects of small particles on fluid turbulence over a flat plate turbulent boundary layer in air have been investigated by Rogers & Eaton [29]. Their measurements clearly demonstrated that the particles suppressed fluid turbulence and showed a strong correlation between the degree of turbulence suppression and particle concentration in the log region of the boundary layer. In this case, one of the principal problems is the determination of the turbulent stress and the turbulent heat flux in the dispersed phase. Melville and Bray [16, 17] have presented a model of two-phase turbulent jet. They have taken the mass fraction of the particles is at most of order unity where as the volume fraction is much less than unity. They have developed a model consisting of a set of differential equations where the mean velocities of the phases are equal. A first-order closure scheme is used and the resulting equations are solved numerically. But they have not developed a model to accommodate the heat transfer analysis. Panda et. al. [25, 26] have studied the turbulent free jet with Suspended Particulate Matter(SPM) by taking the volume fraction and diffusion of particles through the carrier fluid. They have used first-order closure and resulting equations have been solved numerically. They have compared the velocity profile of the carrier fluid with that of the profile given by Schlichting [30]. The computed profile agrees well in the core of the jet but differs outside the core due to the presence of SPM. They have pointed out that the velocity distribution differs from the one that have been computed using the mixing length theory and using the mixing length theory gives a better structure of the turbulent two-phase boundary layer flow. But they have not considered the heat transfer aspects in the modeling effort. Fessler and Eaton [9] have studied experimentally to extend the knowledge gained from turbulence modification. They have observed that moderate mass loadings of small dense particles can reduce the intensity of the gas-phase velocity fluctuations in the channel flow unlike in previous studies. They found that the trend of increasing the attenuation of turbulence with the particle Reynolds number cannot, however, continue indefinitely. But a Reynolds number greater than 400 will enhance the turbulence in the flow.

Understanding the dynamics of a multiphase system has long been an issue of scientific and engineering interest. Of particular interest to this study is the numerical simulation of particle diffusion due to turbulence in the continuous phase, where the density of the particles is assumed to be much greater than the density of the surrounding fluid. The particle laden turbulent jet is also of practical interest because of its presence in a broad range of engineering applications such as combustion, aerosol reaction, jet propulsion and air pollution control. In these processes, the inter-particle collision which is one of the most interesting problems in two-phase flows plays an important role for improving the design of engineering systems and particle transport. Chamkha [7] reported on particulate diffusion effects on the thermal flat plate boundary layer of a two-phase suspension. Chamkha [8] studied the particulate viscous effects on the compressible boundary-layer two-phase flow over a flat plate. Later, Chamkha [6] studied the effects of combined particle-phase diffusivity and viscosity on the compressible boundary layer of a particulate suspension over a flat surface.

In the presence of a solid boundary, the flow behavior and turbulent structure are considerably different from free turbulent flows. In regions close to a solid wall, the structure is dominated by shear due to wall friction and damping of turbulent velocity fluctuations perpendicular to the boundary. This results in a complex flow structure characterized by rapid changes in the mean and fluctuating velocity components concentration within a very narrow region in the immediate vicinity of the wall.

In this correlation, moments of the dispersed-phase velocity fluctuations are directly expressed in terms of the Reynolds stress of the carrier-fluid flow, Ozbelge & Eraslan [24]; for example relations of the gradient type, analogues to the Boussinesq hypothesis in single-phase turbulent stresses in the dispersed phase. In a similar way, for deriving the stress tensor

$$\tau_{ij} = \mu\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

Boussinesq proposed in 1877 that the Reynolds stress be proportional to the strain rate of the mean flow. Thus,

$$\sigma'_{ij} = -\rho\overline{u'_i u'_j} = \rho\epsilon\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right),$$

where $\epsilon$ is called the eddy viscosity. $\epsilon$ is found to be numerically much greater than kinematic viscosity $\nu$.

Using the Prandtl's mixing theory, Schlichting [30], Hinze [12],

$$
\begin{aligned}
\left|\overline{v'}\right| &= const.\left|\overline{u'}\right| = const.l\frac{d\overline{u}}{dy},\\
\left|\overline{v'_p}\right| &= const.\left|\overline{u'_p}\right| = const.l_p\frac{d\overline{u_p}}{dy},\\
\left|\overline{\rho'_p}\right| &= const.l_c\frac{d\overline{\rho_p}}{dy},\\
\left|\overline{T'}\right| &= const.l_t\frac{d\overline{T}}{dy},\\
\left|\overline{T'_p}\right| &= const.l_{t_p}\frac{d\overline{T_p}}{dy},
\end{aligned}
\tag{1}
$$

where $l_p, l_c, l_t$ and $l_{t_p}$ are the mixing lengths for the particle velocity, particle density, fluid temperature and particle temperature, respectively.
Using the Prandtl's mixing hypothesis, Schlichting [30], Hinze [12], the closure equations take the following form:

$$
\begin{aligned}
\overline{u'v'} &= -ll\left|\frac{d\overline{u}}{dy}\right|\frac{d\overline{v}}{dy},\\
\overline{T'v'} &= -l_tl\left|\frac{d\overline{T}}{dy}\right|\frac{d\overline{v}}{dy},\\
\overline{\rho'_pu'} &= -l_cl\left|\frac{d\overline{\rho_p}}{dy}\right|\frac{d\overline{u}}{dy},\\
\overline{\rho'_pv'} &= -l_cl\left|\frac{d\overline{\rho_p}}{dy}\right|\frac{d\overline{v}}{dy},\\
\overline{\rho'_pu'_p} &= -l_cl_p\left|\frac{d\overline{\rho_p}}{dy}\right|\frac{d\overline{u_p}}{dy},\\
\overline{\rho'_pv'_p} &= -l_cl_p\left|\frac{d\overline{\rho_p}}{dy}\right|\frac{d\overline{v_p}}{dy},\\
\overline{u'_pv'_p} &= -l_cl_p\left|\frac{d\overline{u_p}}{dy}\right|\frac{d\overline{v_p}}{dy},\\
\overline{T'_pv'_p} &= -l_{t_p}l_p\left|\frac{d\overline{T_p}}{dy}\right|\frac{d\overline{v_p}}{dy}.
\end{aligned}
\tag{2}
$$

Due to the lack of experimental data on different mixing lengths, all the mixing lengths are assumed approximately equal to each other as these approximations have been used successfully by numerous previous researchers Michaelide & Farmer [18]

$$\begin{aligned}
\overline{u'v'} &= -l^2 \left|\frac{d\overline{u}}{dy}\right| \frac{d\overline{u}}{dy},\\
\overline{T'v'} &= -l^2 \left|\frac{d\overline{T}}{dy}\right| \frac{d\overline{u}}{dy},\\
\overline{\rho_p' u'} &= -l^2 \left|\frac{d\overline{\rho_p}}{dy}\right| \frac{d\overline{u}}{dy},\\
\overline{\rho_p' v'} &= -l^2 \left|\frac{d\overline{\rho_p}}{dy}\right| \frac{d\overline{u}}{dy},\\
\overline{\rho_p' u_p'} &= -l^2 \left|\frac{d\overline{\rho_p}}{dy}\right| \frac{d\overline{u_p}}{dy},\\
\overline{\rho_p' v_p'} &= -l^2 \left|\frac{d\overline{\rho_p}}{dy}\right| \frac{d\overline{u_p}}{dy},\\
\overline{u_p' v_p'} &= -l^2 \left|\frac{d\overline{u_p}}{dy}\right| \frac{d\overline{u_p}}{dy},\\
\overline{T_p' v_p'} &= -l^2 \left|\frac{d\overline{T_p}}{dy}\right| \frac{d\overline{u_p}}{dy}.
\end{aligned} \tag{3}$$

## 2 Mathematical Modeling

In describing a turbulent flow in mathematical form, it is convenient to separate it into a mean motion and into a fluctuation or eddy motion. Denoting the mean quantities by $\overline{u}, \overline{v}, \overline{u_p}, \overline{v_p}$,etc. and fluctuations by $u', v', u_p', v_p'$, etc. respectively, we can write

$$u = \overline{u} + u', v = \overline{v} + v', T = \overline{T} + T', \rho = \overline{\rho} + \rho', p = \overline{p} + p',$$

$$u_j = \overline{u_j} + u_j', u_p = \overline{u_p} + u_p', v_p = \overline{v_p} + v_p', T_p = \overline{T_p} + T_p', \rho_p = \overline{\rho_p} + \rho_p'. \tag{4}$$

The Reynold's forms of boundary layer equations for two-phase flow after introducing the non-dimensional variables

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{\overline{u}}{U}, v^* = \frac{\overline{v}}{U}, u_p^* = \frac{\overline{u_p}}{U}, v_p^* = \frac{\overline{v_p}}{U},$$

$$\rho_p^* = \frac{\overline{\rho_p}}{\rho_{p_0}}, p^* = \frac{\overline{p}}{\rho_0 U^2}, T^* = \frac{\overline{T} - T_\infty}{T_w - T_\infty}, T_p^* = \frac{\overline{T_p} - T_\infty}{T_w - T_\infty}, \tag{5}$$

and after dropping stars can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$u_p \frac{\partial \rho_p}{\partial x} + v_p \frac{\partial \rho_p}{\partial y} = \frac{1}{Re_p} \frac{\partial^2 \rho_p}{\partial y^2} + \frac{\partial}{\partial y} \left(-\overline{\rho_p' v_p'}\right), \tag{7}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - \alpha \frac{1}{1-\varphi} \frac{FL}{U} \rho_p f_p (u - u_p) - \alpha \frac{1}{1-\varphi} \frac{FL}{U} f_p \left(\overline{\rho_p' u'} - \overline{\rho_p' u_p'}\right) + \frac{\partial}{\partial y} \left(-\overline{u'v'}\right), \tag{8}$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{1}{Re_p} \frac{\partial^2 u_p}{\partial y^2} + \frac{FL}{U} f_p (u - u_p) - \frac{1}{\rho_p} \overline{\rho_p' v_p'} \frac{\partial u_p}{\partial y} - \frac{\partial}{\partial y} \left(\overline{u_p' v_p'}\right) - \frac{1}{\rho_p} \overline{u_p' v_p'} \frac{\partial \rho_p'}{\partial y}, \tag{9}$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{1}{Re_p} \frac{\partial}{\partial y} \left[2 \frac{\partial \overline{v}_p}{\partial y} - \frac{2}{3} \left(\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y}\right)\right] + \frac{\partial}{\partial y} \left(-\overline{v_p'^2}\right) + \frac{FL}{U} f_p (v - v_p) - \frac{1}{\rho_p} \overline{\rho_p' v_p'} \frac{\partial \overline{v}_p}{\partial y}, \tag{10}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = -Ec\, u \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) + \frac{1}{Pr.Re} \frac{\partial^2 T}{\partial y^2} + \frac{2\alpha}{3Pr} \frac{1}{1-\varphi} \times \frac{FL}{U} \rho_s f_p (T_p - T) - \frac{\partial}{\partial y} \left(\overline{v'T'}\right) - Ec\, u \frac{\partial}{\partial y} \left(\overline{u'v'}\right) - Ec\, \overline{u'v'} \frac{\partial u}{\partial y}, \tag{11}$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = -\frac{3}{2} Pr Ec u \left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y}\right) + \frac{3}{2} \frac{1}{\alpha \rho_p Re} \frac{\partial^2 T_p}{\partial y^2} - \frac{FL}{U} f_p (T_p - T) - \frac{\partial}{\partial y} \left(\overline{v_p' T_p'}\right) - \frac{3}{2} Pr.Ec\, u_p \frac{\partial}{\partial y} \left(\overline{u_p' v_p'}\right) - \frac{3}{2} Pr.Ec\, \overline{u_p' v_p'} \frac{\partial u_p}{\partial y}, \tag{12}$$

where $f_p$ is the correction factor and is given by, Gharraei et al. [10]

$$f_p = \begin{cases} 1 + 0.15Re_p^{0.687}, & 0 < Re_p \leq 200, \\ 0.914Re_p^{0.282} + 0.0135Re_p, & 200 < Re_p \leq 2500, \\ 0.0167Re_p, & Re_p > 2500, \end{cases} \tag{13a}$$

and the particle Reynolds number,

$$Re_p = \frac{D\,|u_p - u|}{\nu}. \tag{13b}$$

After using the simple algebraic closure proposed by Boussinesq in equations 6 to 12, the governing Reynolds equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{14}$$

$$u_p\frac{\partial \rho_p}{\partial x} + v_p\frac{\partial \rho_p}{\partial y} = \frac{1}{Re_p}\frac{\partial^2 \rho_p}{\partial y^2} + \frac{\partial}{\partial y}\left(l^2\left|\frac{\partial \rho_p}{\partial y}\right|\frac{\partial u_p}{\partial y}\right), \tag{15}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \alpha\frac{1}{1-\varphi}\frac{FL}{U}\rho_p f_p (u - u_p) \\ &\quad + \alpha\frac{1}{1-\varphi}\frac{FL}{U}f_p l^2\left|\frac{\partial \rho_p}{\partial y}\right|\left(\frac{\partial u}{\partial y} - \frac{\partial u_p}{\partial y}\right) \pm l^2\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y} + l^2\left|\frac{\partial u}{\partial y}\right|\frac{\partial^2 u}{\partial y^2}, \end{aligned} \tag{16}$$

$$\begin{aligned} u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} &= \frac{1}{Re_p}\frac{\partial^2 u_p}{\partial y^2} + \frac{FL}{U}f_p (u - u_p) + \frac{1}{\rho_p}l^2\left|\frac{\partial \rho_p}{\partial y}\right|\left(\frac{\partial u_p}{\partial y}\right)^2 \\ &\quad + \frac{\partial}{\partial y}\left(l^2\left|\frac{\partial u_p}{\partial y}\right|\frac{\partial u_p}{\partial y}\right) + \frac{1}{\rho_p}l^2\left|\frac{\partial u_p}{\partial y}\right|\frac{\partial u_p}{\partial y}\frac{\partial \rho_p}{\partial y}, \end{aligned} \tag{17}$$

$$\begin{aligned} u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y} &= \frac{FL}{U}f_p (v - v_p) + \frac{1}{\rho_p}l^2\left|\frac{\partial \rho_p}{\partial y}\right|\frac{\partial u_p}{\partial y}\frac{\partial v_p}{\partial y} - 2l\frac{\partial u_p}{\partial y}\frac{\partial^2 u_p}{\partial y^2} \\ &\quad + \frac{4}{3Re_p}\frac{\partial^2 v_p}{\partial y^2} - \frac{2}{3Re_p}\frac{\partial^2 u_p}{\partial y \partial x}, \end{aligned} \tag{18}$$

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= -Ec\,u\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{1}{PrRe}\frac{\partial^2 T}{\partial y^2} \\ &\quad + \frac{2\alpha}{3Pr}\frac{1}{1-\varphi}\frac{FL}{U}\rho_p f_p (T_p - T) \pm l^2\frac{\partial^2 T}{\partial y^2}\frac{\partial u}{\partial y} \\ &\quad + l^2\left|\frac{\partial T}{\partial y}\right|\frac{\partial^2 u}{\partial y^2} \pm Ec\,u\,l^2\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y} \\ &\quad + Ec\,u\,l^2\left|\frac{\partial u}{\partial y}\right|\frac{\partial^2 u}{\partial y^2} + Ec\,l^2\left|\frac{\partial u}{\partial y}\right|\left(\frac{\partial u}{\partial y}\right)^2, \end{aligned} \tag{19}$$

$$u_p \frac{\partial T_p}{\partial y} + v_p \frac{\partial T_p}{\partial y} = -\frac{3}{2} PrEc\; u_p \left( u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) + \frac{3}{2} \frac{1}{\alpha \rho_p Re} \frac{\partial^2 T}{\partial y^2} - \frac{FL}{U} f_p \left( T_p - T \right) + \frac{\partial}{\partial y} \left( l^2 \left| \frac{\partial T_p}{\partial y} \right| \frac{\partial u}{\partial y} \right) + \frac{3}{2} PrEc\; u_p \frac{\partial}{\partial y} \left( l^2 \left| \frac{\partial u_p}{\partial y} \right| \frac{\partial u_p}{\partial y} \right) - \frac{3}{2} Pr.Ec\; l^2 \left| \frac{\partial u_p}{\partial y} \right| \left( \frac{\partial u_p}{\partial y} \right)^2 . \quad (20)$$

# 3 Method of Solution

To develop a computational algorithm with non-uniform grids, finite-difference expressions are introduced for the various terms in equations 14 to 20 as, Mishra & Tripathy [22] and Tanhehill et al. [33],

$$\frac{\partial W}{\partial x} = \frac{1.5 W_j^{n+1} - 2 W_j^n + 0.5 W_j^{n-1}}{\Delta x} + o\left(\Delta y^2\right), \quad (21)$$

$$\frac{\partial W}{\partial y} = \frac{W_{j+1}^{n+1} - \left(1 - r_y^2\right) W_j^{n+1} - r_y^2 W_{j-1}^{n+1}}{r_y \left(r_y + 1\right) \Delta y_j} + o\left(\Delta y^2\right), \quad (22)$$

$$\frac{\partial^2 W}{\partial y^2} = 2 \frac{W_{j+1}^{n+1} - \left(1 + r_y\right) W_j^{n+1} + r_y W_{j-1}^{n+1}}{r_y \left(r_y + 1\right) \Delta y_j^2} + o\left(\Delta y^2\right), \quad (23)$$

$$\frac{\partial^2 W}{\partial y \partial x} = \frac{1}{\Delta y \Delta x} \left[ 1.5 \left( {u_p}_{j+1}^{n+1} - {u_p}_j^{n+1} \right) - 2 \left( {u_p}_{j+1}^{n} - {u_p}_j^{n} \right) + 0.5 \left( {u_p}_{j+1}^{n-1} - {u_p}_j^{n-1} \right) \right], \quad (24)$$

$$W_j^{n+1} = 2 W_j^n - W_j^{n-1} + o\left(\Delta x^2\right), \quad (25)$$

and

$$y_{j+1} - y_j = r_y \left( y_j - y_{j-1} \right) = r_y \Delta y_j, \quad (26)$$

where $W$ stands for either $u$ or $u_p$ or $v_p$ or $T$ or $T_p$ or $\rho_p$. Here a general three-point representation of $\frac{dW}{dy}$ on a non-uniform grid that produces the smallest truncation error is used. In this way, equation 14 reduces into the difference equation

$$v_j^{n+1} = v_{j-1}^{n+1} - \frac{1}{2} \frac{\Delta y}{\Delta x} \times \left( 1.5 u_j^{n+1} - 2 u_j^n + 0.5 u_j^{n-1} + 1.5 u_{j-1}^{n+1} - 2 u_{j-1}^n + 0.5_{j-1}^{n-1} \right), \quad (27)$$

and each of the equations 15 to 20 reduces to a form,

$$a_j W_{j-1}^{n+1} + b_j W_j^{n+1} + c_j W_{j+1}^{n+1} = d_j, \quad j = 2 \text{ to } j_{max} - 1. \tag{28}$$

## 4 Boundary and Initial Conditions

Due to the no-slip condition at the wall, the wall boundary conditions of the gaseous phase are given by

$$u = v = 0, \quad T = T_w. \tag{29}$$

On the other hand, the particles may be in a slip motion at the wall. Assuming that the particle mass is concentrated pointy at the center of the particle; particles cannot exist in the region within the distance from the wall smaller than the radius of the particle. Therefore, the velocity, temperature and particle density for the particulate phase at the wall may be approximated as follows:

$$u_p = u_{pw}(x), \; v_p = 0, \; T_p = T_{pw}(x), \; \rho_p = \rho_{pw}(x). \tag{30}$$

By using the non-dimensional quantities 5, equations 14 to 20 will be solved subjected to the boundary conditions

$$y = 0 \; : \; u = 0, \; v = 0, \; u_p = u_{pw}(x), \; \rho_p = \rho_{pw}(x), \; T = 1, T_p = T_{pw}(x), \tag{31}$$

$$y = \infty \; : \; u = u_p = U(x), \; \rho_p = 1, \; v_p = 0, \; T = 0, \; T_p = 0. \tag{32}$$

The initial value of velocity, density, and temperature of the particle phase on the wall can be calculated by Mishra & Tripathy [22] as

$$u_{pw} = -\frac{2}{3}\frac{FL}{U}\Delta x + \frac{4}{3}u_{p_1}^{n} - \frac{1}{3}u_{p_1}^{n-1}, \tag{33}$$

$$\rho_{pw} = \frac{2\rho_{p_1}^{n} - 0.5\rho_{p_1}^{n-1}}{1.5 - \frac{\frac{FL}{U}\Delta x}{u_{p_1}^{n+1}}}, \tag{34}$$

$$T_{pw} = \frac{2T_{p_1}^{n} - 0.5T_{p_1}^{n-1} + \frac{FL}{U}\frac{\Delta x}{u_{p_1}^{n+1}}}{1.5 + \frac{FL}{U}\frac{\Delta x}{u_{p_1}^{n+1}}}. \tag{35}$$

The shear stress coefficient and the wall heat transfer rate for the fluid phase can be calculated as

$$c_f = \frac{2}{U^2\sqrt{Re}}\left[\frac{u_3^{n+1} - \left(1 - r_y^2\right)u_2^{n+1} - r_y^2 u_1^{n+1}}{r_y\left(1 + r_y\right)\Delta y}\right], \tag{36}$$

$$Nu = -\sqrt{Re}\left[\frac{T_3^{n+1} - \left(1 - r_y^2\right)T_2^{n+1} - r_y^2 T_1^{n+1}}{r_y\left(1 + r_y\right)\Delta y}\right]. \tag{37}$$

# 5 Computational Results and Discussion

In this problem, the basic features of the gas-particulate thermal boundary layer flow over a semi-infinite flat plate have been studied numerically by employing the finite difference technique.We choose the following values of the various parameters involved.
$U = 60.96, 160.96, 260.96, 360.96; L = 3.048; Re_p = 1.0 \times 10^{-04};$
$\rho = 0.94; Ec = 0.1; \mu = 21.85 \times 10^{-06}; \varphi = 1.01 \times 10^{-05};$
$\epsilon = 0.05, 0.1, 0.2; D = 100, 50\mu m; \rho_s = 800, 2403, 8010; Pr = 0.71, 1.0, 7.0$
From some typical outputs of the programme for the carrier fluid without SPM, we conclude that the result for $r_y = 1.110$ which gives the least r.m.s. error. Therefore the result for $r_y = 1.110$ is accepted and used for the physical interpretation of the result. Similarly, the computational results for the flow of fluids with SPM are also obtained. It is observed that the result obtained for $r_y = 1.680$ is acceptable as the r.m.s. error is least. Fig. 1 shows the laden and un-laden fluid mean velocity profile at both $x = 26$ locations downstream of the boundary layer. The free stream velocity at the leading edge was set to be the same in both laden and un-laden flows. The comparison of the single-phase and two-phase flows mean velocities shows that there exists a decrease in magnitude of the carrier fluid mean fluid velocity inside the boundary layer by the particles. That is, the two-phase interaction will decrease the velocity gradient inside the boundary layer, and this fact leads to decreases in the skin friction coefficient and the surface heat transfer rate (shown in Fig. 13 and Table 1). Qualitatively, this result has many similarities to that documented by Rogers & Eaton [29]. This produces a turbulent suppression by the particles in the region of the boundary layer. This suppression is most likely a result of the drag loading, particle initial conditions, fluid length scale and the particle/wall interactions. The velocity and temperature profiles of the carrier fluid are shown in Fig. 2.

In the analysis, we have examined the effects on the flow properties by varying the particle diameter for fixed values of the other parameters.

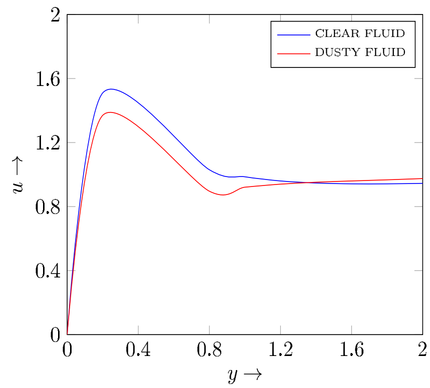


Figure 1: Velocity distribution ($u$) of carrier fluid with and without SPM.

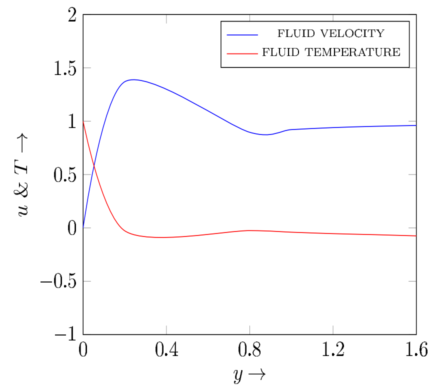


Figure 2: Variation of velocity ($u$) and Temperature(T) distribution of carrier fluid.

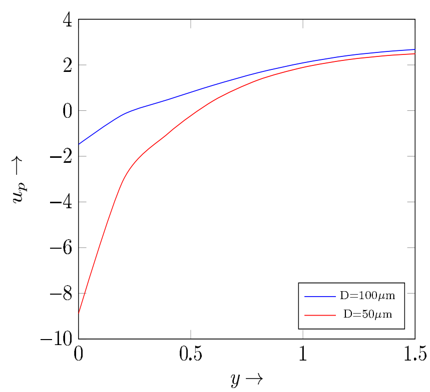


Figure 3: Variation of particle velocity ($u_p$) with diameter(D) of particles.

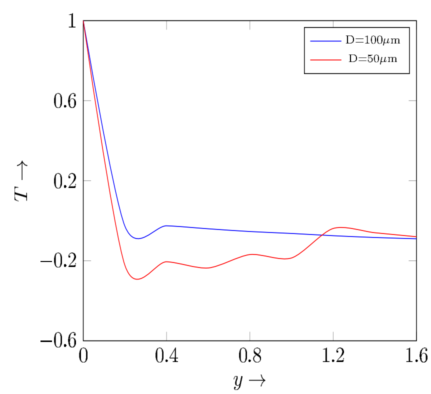


Figure 4: Variation of temperature ($T$) of carrier fluid with diameter ($D$) of the particles.

This is equivalent to examining the effects of varying the total surface area of the particles by fixing the volume fraction but changing their number. Doubling the particle diameter while fixing the volume fraction reduces the number of particles by a factor of 8 and the total surface area by a factor of 2 and hence reduces the two-way coupling effects. It is clear from Fig. 3 that the particle-phase mean velocity profile for various sizes of the particles. It is observed that the larger particles flow faster than that of the smaller ones near the wall. From Fig. 4, it is concluded that the presence

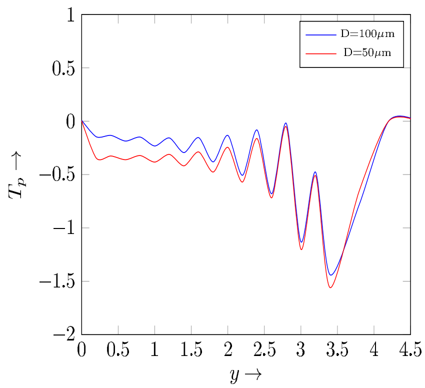


Figure 5: Variation of temperature ($T_p$) of particle phase with diameter ($D$) of particles.

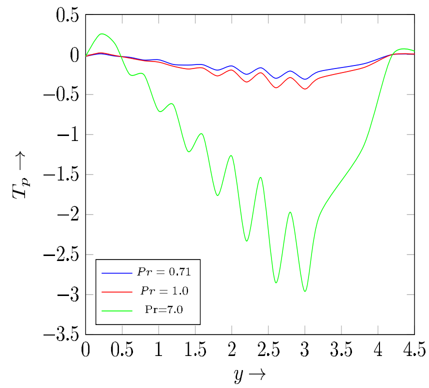


Figure 6: Variation of temperature ($T_p$) of particle phase with Prandtl number ($Pr$).

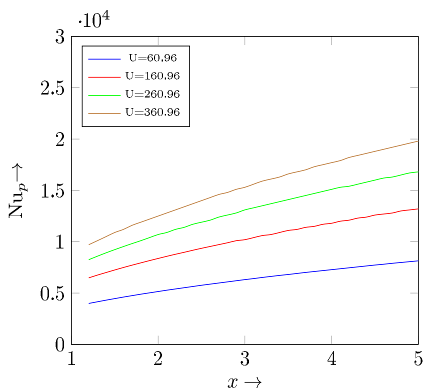


Figure 7: Variation of particle Nusselt number ($Nu_p$) with free stream velocity ($U$).

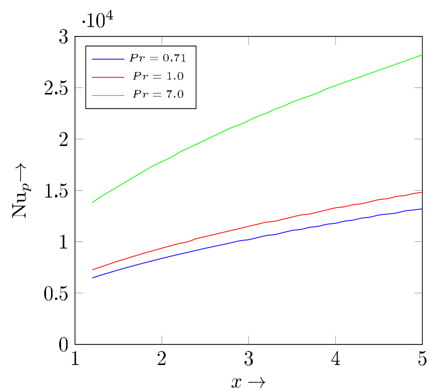


Figure 8: Variation of particle Nusselt number ($Nu_p$)with Prandtl number ($Pr$).

of the larger particles increases the mean temperature of the carrier fluid than that of the presence of small particles inside the boundary layer. Fig. 5 depicts the temperature profile of the dispersed phase for various sizes of the particles. It is observed that the mean temperature of the dispersed phase with smaller particles is less than that of the dispersed phase with larger particles. The variation of the particle-phase temperature with the Prandtl number (Pr) is shown in the Fig.6. In general, it is predicted that the particle-phase temperature decreases as the Prandtl number increases.

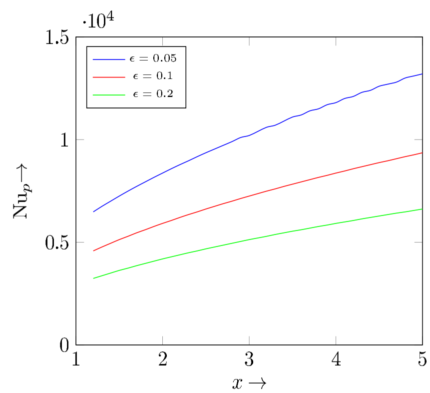


Figure 9: Variation of particle Nusselt number ($Nu_p$) with diffusion parameter($\epsilon$)

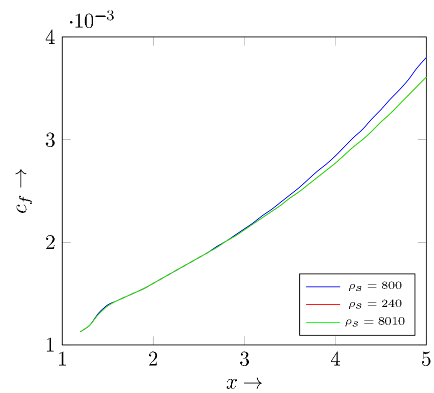


Figure 10: Variation of skin friction ($c_f$) with material density($\rho_s$) of particles

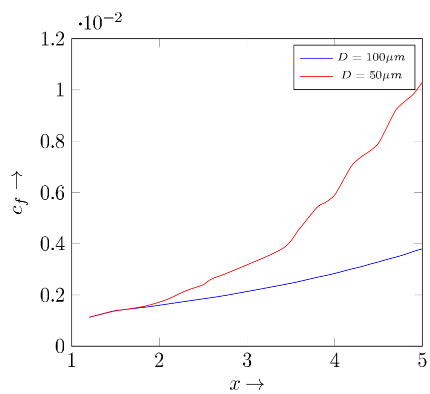


Figure 11: Variation of skin friction ($c_f$) with diameter($D$) of particles

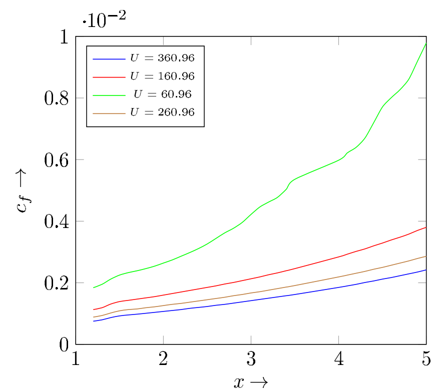


Figure 12: Variation of skin friction ($c_f$) with free stream velocity ($U$)

The variation of the particle-phase Nusselt number ($Nu_p$) calculated on the basis of the formula given by Han et al. [11],

$$Nu_p = 2 + 0.6Re_p^{0.5}Pr^{0.33} \tag{38}$$

with the free stream velocity($U$), Prandtl number($Pr$), the diffusion parameter ($\epsilon$) are presented in Figs. 7, 8 and 9. The magnitude of $Nu_p$ goes on increasing with the increase of $U$ and $Pr$ for fixed values of the other parameters but decreases with the increase of the diffusion parameter in-

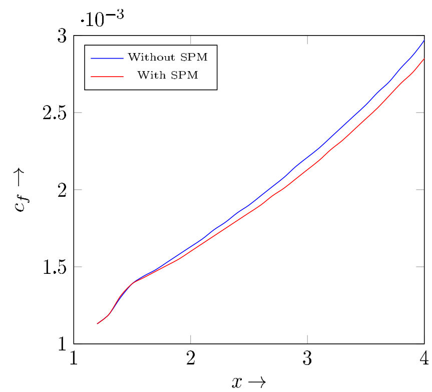


Figure 13: Variation of skin friction $(c_f)$ with & without SPM

dicating that the heat transfer from the particle phase to the fluid phase is more in case of larger values of the parameters $U$ and $Pr$ but less in case when the diffusion of particles is more. Fig. 10 shows the response of the skin friction coefficient in the presence of particles of different material density and clearly, it depicts a decrease in magnitude of the skin friction coefficient as the material density of the particles increases. The presence of smaller particles increases the skin friction coefficient, which is observed from Fig. 11. Fig. 12 depicts that the magnitude of the skin friction coefficient goes on decreasing with the increase of the free stream velocity$(U)$. From which we conclude that a larger free stream velocity stabilizes the flow and the turbulent character is diminished. Finally, we draw a conclusion from Fig. 13 that the presence of SPM decreases the magnitude of the skin friction coefficient and also from Table 1, it is concluded that the presence of SPM decreases the heat transfer from the plate to the fluid towards the leading edge of the plate whereas it increases the heat transfer towards the far down stream of the plate.

# 6 Conclusion

In this paper, a two-dimensional, turbulent boundary layer flow with SPM is numerically simulated. An algorithm based on the Eulerian-Eulerian approach was developed, and the governing Reynolds forms of the equations were solved by using a finite-difference technique with non-uniform grids. The results obtained in this work were found to be in agreement with those of previous investigations. It was found that the heat transfer from the

Table 1: Comparison of fluid Nusselt number($Nu$) with and without SPM

| x | Nu without SPM | Nu with SPM |
|---|---|---|
| 1.20 | 6.67E+02 | 2.86E+02 |
| 1.40 | 7.28E+02 | 3.38E+02 |
| 1.60 | 7.57E+02 | 3.57E+02 |
| 1.80 | 7.70E+02 | 3.77E+02 |
| 2.00 | 7.88E+02 | 3.86E+02 |
| 2.20 | 8.10E+02 | 3.76E+02 |
| 2.40 | 8.32E+02 | 3.12E+02 |
| 2.50 | 8.46E+02 | 1.03E+02 |
| 3.00 | 5.66E+04 | 4.69E+04 |
| 3.50 | 4.00E+03 | 5.24E+03 |
| 4.00 | 1.46E+03 | 4.83E+02 |
| 4.50 | 1.44E+03 | 2.07E+03 |
| 5.00 | 1.40E+03 | 2.07E+03 |

particle phase to the fluid phase was higher for larger values of the free stream velocity and Prandtl number but it was predicted to be lower in the case when the diffusion of particles increased. Also, it was predicted that a larger free stream velocity stabilized the flow and that the turbulent character of the flow was diminished. In addition, it was concluded that the presence of suspended particulate material damped the fluid turbulence and caused reductions in both the skin friction and the heat transfer.

## References

[1] G.K. Batchelor, *The dynamics of a mixture of two components.* In:10th Australian Fluid mechanics Conferences, University of Melbourne,(1989).

[2] G.K. Batchelor, *The effect of Brownian motion on the bulk stress in a suspension of spherical particles*, J. Fluid Mech., **83**(1977) 97-117.

[3] S.S. Bishoyi, A.R. Sahu, S.K. Mishra, *CFD modeling of two-phase flow with lift force*, Int. J. Pure Appl. Math. Sci., **5(1)** (2012) 99-113.

[4] R.G. Boothroyd, *Flowing gas-solids suspensions*, Chapman & Hall, London, (1971).

[5] T. Cebeci, A.M.O. Smith, *Analysis of turbulent boundary layers*, Academic Press, New York, (1974).

[6] A.J. Chamkha, *Effect of combined particle-phase diffusivity and viscosity on the compressible boundary layer of a particulate suspension over a flat surface*, J. Heat Transfer, **121** (1999) 420-429.

[7] A.J. Chamkha, *Effects of particulate diffusion on the thermal flat plate boundary layer of a two-phase suspension*, J. Heat Transfer, **116** (1994) 236-239.

[8] A.J. Chamkha, *Particulate viscous effects on the compressible boundary-layer two-phase flow over a flat plate*. Int. Commun. Heat Mass Transfer, **25** (1998) 279-288.

[9] J.R. Fessler, J.K. Eaton, *Turbulence modification by particles in a backward-facing step flow*, J. Fluid Mech., **394** (1999) 97-117.

[10] R. Gharraei, E. Esmaeilzadeh, S. Baheri, *Evaluation of two-phase turbulence closure models modifications in near wall region of boundary layer*. In: 16th Australian Fluid Mechanics Conference, Crown Plaza, Gold Coast, (2007) 1033-1037.

[11] K.S. Han, H.J. Sung, M.K. Chung, *Analysis of heat transfer in a pipe carrying two-phase gas-particle suspension*, Int. J. Heat Mass Transfer, **34(1)** (1991) 69-78.

[12] J.O. Hinze, *Turbulence, McGraw Hill, New York*, (1954).

[13] A. Hossain, J. Naser, *Analytical and CFD investigation of turbulent diffusion model for particle dispersion and deposition in horizontal pipe flow*, J. Mech. Eng., **40(1)** (2009) 39-53.

[14] K.B. Lee, M.K. Chung, *Refinement of mixing length-model for prediction of gas-particle flow in a pipe*, Int. J. Multiphase Flow, **13** (1987) 275-282.

[15] F.E. Marble, *Dynamics of a gas combining small solid particles*, In:Proceedings of the 5th AGARD Combustion and Propulsion Symposium, (Pergamon Press) New York,(1963) 175-215.

[16] W.K. Melville, K.N.C. Bray, *A model of the two-phase turbulent jet*, Int. J. Heat Mass Transfer, **22** (1979b) 647-656.

[17] W.K. Melville, K.N.C. Bray, *The two-phase turbulent jet*, Int. J. Heat Mass Transfer, **22** (1979a) 279-287.

[18] E.E. Michaelides, L.K. Farmer, *A model for slurry flows based on the equation of Turbulence*. J. Pipelines,**4** (1984) 185-191.

[19] S.K. Mishra, S.S. Bishoyi, *Numerical modeling of combined free and forced convection flow with SPM past a vertical plate*, Acta Cienc. Indica Math.,**38M(2)** (2012) 301-315.

[20] S.K. Mishra, S.S. Bishoyi, *Numerical modeling of free convection two-phase flow past a vertical plate*, Far East J. Appl. Math., **59(2)** (2011) 75-103.

[21] S.K. Mishra, P.K. Tripathy, *Approximate solution of two phase thermal boundary layer flow"*, Reflections des ERA, **6 (2)** (2011a) 113-148.

[22] S.K. Mishra, P.K. Tripathy, *Mathematical and numerical modeling of two phase flow and heat transfer using non-uniform grid*, Far East J. Appl. Math., **54(2)** (2011b) 107-126.

[23] G. Ooms, P. Poesio, *Effects of particle inertia and gravity on the turbulence in a suspension*, Phys. Fluids, **17** (2005) 125101-1 to 125101-12.

[24] T.A. Ozbelge, A.N. Eraslan, *A computational hydrodynamics and heat transfer study in turbulent up- flows of dilute slurries through a concentric annulus*, Turkish J. Eng. Environ. Sci., **30** (2006) 1-13.

[25] T.C. Panda, S.K. Mishra, A.K. Das, *Modeling the turbulent diffusion of suspended particulate matter (SPM) in Ekman boundary layer based on first order closure*, Acta Cienc. Indica Math., **25M(4)** (1999) 365-374.

[26] T.C. Panda, S.K. Mishra, A.K. Das, *Turbulent free jet with suspended particulatematter(SPM)*, Proc. Natl. Acad. Sci. India,**74(A)-III** (2004) 297-310.

[27] A.J. Reynolds, *The prediction of turbulent Prandtl and Schmidt Numbers*, Int. J. Heat Mass Transfer, **18** (1975) 1055-1069.

[28] M.A. Rizk, S.E. Elghobashi, *The motion of a spherical particle suspended in a turbulent flow near a plane wall*, Phys. Fluids, **28** (1985) 806-817.

[29] C.B. Rogers, J.K. Eaton, *The effect of small particles on fluid turbulence in a flat - plate, turbulent boundary layer in air*, Phys. Fluids A, **3(5)** (1991) 928-937.

[30] H. Schlichting, *Boundary Layer Theory*, 7th edition, (1979) 578-583.

[31] B. Shotorban, S. Balachandar, *Particle concentration in homogeneous shear turbulence simulated via Lagrangian and equilibrium Eulerian approaches*, Phys. Fluids, **18** (2006) pp.065105.

[32] S.L. Soo, *Boundary layer motion of a gas solid suspension"*, University of Illinois, Project squid Report-ILL-3p, (1961).

[33] J.C. Tannehill, D.A. Anderson, R.H. Pletcher, *Computational fluid mechanics and heat transfer*, Taylor & Francis, 2nd edition,(1997), 53, 278.

[34] L.B. Torobin, W.H. Gauvin, *Fundamental aspects of solid-gas flow*, Can. J. Chem. Eng., **37** (1959) 129-141.

[35] P.K. Tripathy, S.S. Bishoyi, S.K. Mishra, *Numerical investigation of two-phase flow over a wedge*, Int. J. Numer. Methods Appl., **8(1)** (2012a) 45-62.

[36] P.K. Tripathy, S.K. Mishra, *Two-phase thermal boundary layer flow*, Int. J. Eng. Res. Technol., **1(8)** (2012b) 1-10.

[37] P.K. Tripathy, A.R. Sahu, S.K. Mishra, *Numerical modeling of two phase flow with transverse force"*, Acta Cienc. Indica Math., **38M(4)** (2012c) 675-686.