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Iterative identification algorithm for tumor model using controlled ARMA model

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Abstract. Since system identification of a tumor model is a primary need for controlling tumor model system, accessing suitable and applicable identification methods is a necessary object. In this paper, firstly, for estimating controlled auto-regressive moving average (CARMA) systems, two identification methods, namely generalized projection algorithm (GPA) and two-stage GPA (2S-GPA), are introduced and presented in order to estimate unknown parameters of a specific and vital tumor model. Furthermore, effectiveness of such methods, like convergence rate and estimation error, are discussed and considered. The introduced algorithms are simulated to prove these methods effectiveness, and data derived from the simulations are depicted through tables and figures.

Keywords: Generalized projection algorithms, two-stage identification, system identification, parameter estimation.
AMS Subject Classification 2010: 39A05,37-04.

1 Introduction

In recent years, numerical methods have gained a plethora of attention for solving matrix equations [14, Page 1] [9, Page 43] [3, Page 1] [6, Page 1] [5, Page 1-2] parameter estimation, and filtering problems [17, Page 3], [23, Page 3], [8, Page 1]. Parameter estimation methods also use signal modeling [20, Page 1] and process control [21, Page 1-2]. The iterative identification approaches benefit from making ample usage of all output input information and enhancing the system identification precision [19, Page 1], [22, Page 1]. In control engineering, many multi-variable systems having complicated configurations and disturbances have uncertainty, including multi-input multi-output systems (MIMO), multiple input single output (MISO) systems and single input multiple output (SIMO) systems.

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Many efforts have been made in the realm of CARMA system identification [15, Page 1]. In the literature, Raja et al. presented a two stage least mean square fractional identification method for system idenification of a typical CARMA system [15, Page 3]. In [13, Page 4], nature-inspired heuristic set of rules for system identification of the CARMA system is introduced by Mehmood et al. For Hammerstein nonlinear CARMA systems, filtering-based least-squares system identification approaches rendered and discussed in [12, Page 2] by Mao et al.. In [11, Page 4], Hu et al, introduced two novel GPA for estimating stochastic systems and auto-regressive moving average (ARMA) systems.

In 2020, new cancer cases rate and cancer death rate was 19.3 million and 10.0 million per yea [18, Page 1]. So, finding a solution to tackle the problem of curing cancer is an integral issue [16, Page 1], [2, Page 1] [7, Page 1]. In order to cure cancer, accessing a suitable mathematical model is a priority [1, Page 1],

In [4, Page 3], an appropriate mathematical model is presented, and it was shown that optimal control therapy is more efficient than traditional pulsed chemotherapy. In order to control the tumor model, either by optimal control, adaptive control, or other means, having an accurate estimation of the system's parameters is a necessity. In this paper, we aim to identify a study case derived at [4, Page 3].

Furthermore, since there has been no research done in estimating CARMA system parameters, by taking advantage of generalized projection algorithms, in this contribution, our goal is to render two novel approaches for estimating parameters of a CARMA tumor model by direct use of GPA. At first, we introduce the mathematical terms of a generic CARMA model system. Afterward, the generalized projection methods are introduced mathematically, and two algorithms are depicted step-by-step. The step by step algorithms are represented in a simple frame for the reader to simplify them for further use. Finally, the effectiveness of presented algorithms for estimation of tumor model system is shown and brought up. Novelties of this paper are listed as follows:

1. Mathematical proof of generalized projection algorithm (GPA) and two-stage GPAs algorithms (2S-GPAs) for CARMA systems;
2. Introduction of GPA for CARMA systems;
3. Introduction of 2S-GPA for CARMA systems;
4. Showing effectiveness and convergence of the introduced algorithms for estimating CARMA system parameters;
5. Identification of parameters of a particular tumor model system.

The rest of the contribution is shaped as follows: In the following section, a nuance characteristic of the system configuration regarding with the CARMA configuration is brought up. Also Section 2 includes the mathematics of two novel GPA algorithm. Section 3 describes a specific tumor model. In Section 4, all the necessary simulations for showing the effectiveness of new algorithms are illustrated by identifying a tumor model. Eventually, in Section 5, all the conclusions are derived.

2 System configuration and novel identification algorithms

Take the following controlled auto-regressive moving average (CARMA) system into consideration

$$A(q)y(t) = B(q)u(t) + D(q)v(t). \tag{1}$$

Here $u(t)$ and $y(t)$ are the sequence of input and output of the system respectively and $v(t)$ is a sequence of white noise with variance σ^2 and zero mean. Also $A(q)$, $B(q)$, and $D(q)$ are the polynomials in the shift operator of unit backward , i.e., $q^{-1}u(t) = u(t-1)$. For simplicity in the rest of the paper, we have the following notations: $A := X$, indicates A is determined as X; I (I_n), is identity matrix of suitable size $(n \times n)$; 1_n, is a vector of n-dimensional with all elements equal to 1. The superscript T represents the transpose of matrix and the norm of matrix X is determined by $\|X\|^2 = tr(XX^T)$.

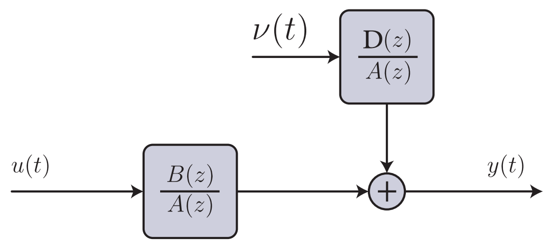


Figure 1: A system explained by configuration of CARMA.

According to the CARMA system illustrated in Figure 1, we define $A(q)$, $B(q)$, and $D(q)$ as polynomials of specific orders which are n_a, n_b, n_d, respectively. On the other hand, we have

$$A(q) := 1 + \sum_{i=1}^{n_a} a_i q^{-i}, \tag{2}$$

$$B(q) := \sum_{i=1}^{n_b} b_i q^{-i}, \tag{3}$$

$$D(q) := 1 + \sum_{i=1}^{n_d} d_i q^{-i}. \tag{4}$$

Let $n := n_a + n_b + n_d$, the following parameters are defined:

$$\Theta := \begin{bmatrix} \theta \\ \vartheta \end{bmatrix} \in \mathbb{R}^n, \tag{5}$$

$$\theta := [a_1, a_2, \ldots, a_{n_a}, b_1, b_2, \ldots, b_{n_b}]^T \in R^{n_a+n_b}, \tag{6}$$

$$\vartheta := [d_1, d_2, \ldots, d_{n_d}]^T \in R^{n_c}. \tag{7}$$

Without loss of generality with the assumption $y(t) = 0$, $u(t) = 0$ and $v(t) = 0$ for $t \leq 0$, the corresponding vectors of information are considered:

$$\varphi(t) := \begin{bmatrix} \phi(t) \\ \psi(t) \end{bmatrix} \in \mathbb{R}^n, \tag{8}$$

$$\phi(t) := \begin{bmatrix} -y(t-1), & -y(t-2), & \ldots, & -y(t-n_a), & u(t-1), & u(t-2), & \ldots, & u(t-n_b) \end{bmatrix}^T \in \mathbb{R}^{n_a+n_b}, \tag{9}$$

$$\psi(t) := \begin{bmatrix} v(t-1), & v(t-2), & \ldots, & v(t-n_d) \end{bmatrix}^T \in \mathbb{R}^{n_d}. \tag{10}$$

According to the equation (1), the following identification model is considered

$$
\begin{aligned}
y(t) =&[1 - A(q)]y(t) + B(q)u(t) + D(q)\nu(t)\\
=&(-a_1q^{-1} - a_2q^{-2} - \cdots - a_{n_a}q^{-n_a})y(t) + (b_1q^{-1} + b_2q^{-2} + \cdots + b_{n_b}q^{-n_b})u(t)\\
&+ (1 + d_1q^{-1} + d_2q^{-2} + \cdots + d_{n_d}q^{-n_d})\nu(t)\\
=& - a_1y(t-1) - a_2y(t-2) - \cdots - a_{n_a}y(t-n_a)\\
&+ b_1u(t-1) + b_2u(t-2 + \cdots + b_{n_b}u(t-n_b)\\
&+ \nu(t) + d_1\nu(t-1) + d_2\nu(t-2) + \cdots + d_{n_d}\nu(t-n_d)\\
=&[-y(t-1), -y(t-2), \ldots, -y(t-n_a), +u(t-1), +u(t-2), \ldots, +u(t-n_b)]\theta\\
&+ [\nu(t-1), +\nu(t-2), \ldots, +\nu(t-n_d)]\vartheta + \nu(t).
\end{aligned}
$$

In summary we have

$$
\begin{aligned}
y(t) &= \phi^T(t)\theta + \psi^T(t)\vartheta + \nu(t), && (11)\\
&= \varphi^T\Theta + \nu(t). && (12)
\end{aligned}
$$

Now based on CARMA system configuration we present two novel parameter estimation method.

2.1 Generalized projection algorithm

Acoording to the system model presented in equation (12), a gradient criterion function is determined as

$$J_1(\Theta) = \frac{1}{2}[y(t) - \varphi^T\Theta]^2.$$

Now we define $\mu(t) \geq 0$ which is the scale of step or the factor of convergence . Utilizing the negative gradient quest method to make $J_1(\Theta)$ minimum, the following gradient identification approach is defined

$$
\begin{aligned}
\hat{\Theta}(t) &= \hat{\Theta}(t-1) - \mu(t)\nabla[J_1(\hat{\Theta})(t-1)],\\
&= \hat{\Theta}(t-1) + \mu(t)\varphi(t)[y(t) - \varphi^T(t)\hat{\Theta}(t-1)], \qquad (13)\\
&= \hat{\Theta}(t-1) + \mu(t)\varphi(t)e(t).
\end{aligned}
$$

and $e(t)$ is determined as

$$e(t) := y(t) - \varphi^T(t)\hat{\Theta}(t-1).$$

By replacing $\Theta = \hat{\Theta}(t)$, into $J_1(\Theta)$, we have

$$
\begin{aligned}
J_1(\hat{\Theta}(t)) &= \frac{1}{2}\Big(y(t) - \varphi^T(t)[\hat{\Theta}(t-1) + \mu(t)\varphi(t)e(t)]\Big)^2,\\
&= \frac{1}{2}\Big(y(t) - \varphi^T(t)\hat{\Theta}(t-1) - \mu(t)\|\varphi(t)\|^2e(t)\Big)^2,\\
&= \frac{1}{2}\Big(e(t) - \mu(t)\|\varphi(t)\|^2e(t)\Big)^2,\\
&= \frac{1}{2}\Big(1 - \mu(t)\|\varphi(t)\|^2\Big)^2e^2(t).
\end{aligned}
$$

Considering $e(t) \neq 0$, the most relevant choice in order to minimize $J_1(\hat{\Theta}(t))$ is

$$\mu(t) = \frac{1}{\|\varphi(t)\|^2}. \tag{14}$$

By taking advantage of the above equation, we can gain the projection algorithm for estimation of vector of parameter Θ of the CARMA configuration in equation (12), as follow.

$$\hat{\Theta}(t) = \hat{\Theta}(t-1) + \frac{\varphi(t)}{\|\varphi(t)\|^2}[y(t) - \varphi^T(t)\hat{\Theta}(t-1)]. \tag{15}$$

In order to avoid singularity in the above equation, following relation will be used

$$\hat{\Theta}(t) = \hat{\Theta}(t-1) + \frac{\varphi(t)}{1+\|\varphi(t)\|^2}[y(t) - \varphi^T(t)\hat{\Theta}(t-1)]. \tag{16}$$

Note that $\hat{\Theta}(0) = p_0^{-1} I_n$, in which p_0 is a preset and selective value which could be in order of 10^6. So as to decrease the sensibility of the projection approach to noise, it is better to alter the vector of gain. Imagine q is the length of data , and by defining a novel variable $r(t)$, that will be increased by the rise in the q and is defined as

$$r(t) := \|\varphi(t)\|^2 + \|\varphi(t-1)\|^2 + \cdots + \|\varphi(t-q)\|^2.$$

and could be represented as

$$r(t) = r(t-1) + \|\varphi(t)\|^2 + \|\varphi(t-q)\|^2.$$

By considering $\mu(t) = \frac{1}{r(t)}$, the vector of gain $L(t) := \frac{\varphi(t)}{\|\varphi(t)\|^2} \in \mathbb{R}^n$, or $L(t) := \frac{\varphi(t)}{1+\|\varphi(t)\|^2} \in \mathbb{R}^n$, could be altered as follows

$$L(t) = \frac{\varphi(t)}{r(t)} = \frac{\varphi(t)}{r(t-1) + \|\varphi(t)\|^2 + \|\varphi(t-q)\|^2}.$$

In brief, considering the above equations, the GPA for identifying Θ has the following set of formulations:

$$\hat{\Theta}(t) = \hat{\Theta}(t-1) + \frac{\varphi(t)}{r(t)} e(t), \text{ where } \hat{\Theta}(0) = p_0^{-1} I_n, p_0 \sim 10^6, \tag{17}$$

$$e(t) = y(t) - \varphi^T(t)\hat{\Theta}(t-1), \tag{18}$$

$$r(t) = r(t-1) + \|\varphi(t)\|^2 + \|\varphi(t-q)\|^2, \quad r(0) = 1, \tag{19}$$

$$\varphi(t) = [\phi^T(t), \psi^T(t)]^T, \tag{20}$$

$$\phi(t) = [-y(t-1), -y(t-2), \ldots, -y(t-n_a), u(t-1), u(t-2), \ldots, u(t-n_b)]^T, \tag{21}$$

$$\psi(t) = [v(t-1), v(t-2), \ldots, v(t-n_d)]^T, \tag{22}$$

$$\hat{\Theta}(t) = [\hat{\theta}^T(t), \hat{\vartheta}^T(t)]^T, \tag{23}$$

$$\hat{\theta}(t) = [\hat{a}_1(t), \hat{a}_2(t), \ldots, \hat{a}_n(t), \hat{b}_1(t), \hat{b}_2(t), \ldots, \hat{b}_n(t)]^T, \tag{24}$$

$$\hat{\vartheta}(t) = [\hat{d}_1(t), \hat{d}_2(t), \ldots, \hat{d}_n(t)]^T. \tag{25}$$

The above equations introduced for the GPA used for the CARMA systems, are presented in Algorithm 1 and the flowchart is provided in Figure 2.

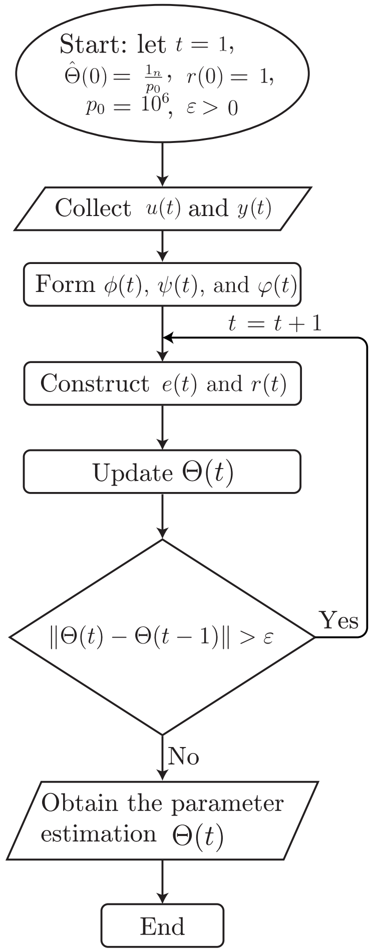


Figure 2: GPA flowchart

Algorithm 1 GP algorithm.

$t \leftarrow 1$
$\hat{\Theta}(0) \leftarrow \frac{1_n}{p_0}$
$r(0) \leftarrow 1$
$p_0 \leftarrow 10^6$
$\varepsilon \leftarrow$ small positive value
$u(t) \leftarrow$ collect input data
$y(t) \leftarrow$ collect output data
Form $\phi(t)$, $\psi(t)$, and $\varphi(t)$ using equations (20-22)
while $\|\Theta(t) - \Theta(t-1)\| > \varepsilon$ **do**
    Compute $e(t)$ and $r(t)$ using (18) and (19)
    Update $\Theta(t)$ according to (17)
    $t \leftarrow t + 1$
end while

2.2 Two stage generalized projection algorithm

In this part of the paper the 2S-GP algorithm is introduced. In the following, two intermediate output variables are defined

$$y_1(t) := y(t) - \psi^T(t)\vartheta, \tag{26}$$

$$y_2(t) := y(t) - \phi^T(t)\theta. \tag{27}$$

Now the system in equation (11) is separated into two imaginary subsystems

$$y_1(t) = \phi^T(t)\theta + v(t), \tag{28}$$

$$y_2(t) = \psi^T(t)\vartheta + v(t). \tag{29}$$

In order to tend $y(t)$ to $\varphi^T - \Theta$, the following cost functions are defined and tried to be minimized.

$$J_2(\theta) = \frac{1}{2}\big(y_1(t) - \phi^T(t)\theta\big)^2, \tag{30}$$

$$J_3(\vartheta) = \frac{1}{2}\big(y_2(t) - \psi^T(t)\vartheta\big)^2. \tag{31}$$

Similar to GPA, by utilizing the negative gradient quest method the criterion functions $J_2(\theta)$ and $J_3(\vartheta)$ are minimized and following gradient-based recursive relations are obtained

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\phi(t)}{1+\|\phi(t)\|^2}[y_1(t) - \phi^T(t)\hat{\theta}(t-1)], \text{ where } \hat{\theta}(0) = \frac{1_{n_{a+b}}}{p_0}, \tag{32}$$

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + \frac{\psi(t)}{1+\|\psi(t)\|^2}[y_2(t) - \psi^T(t)\hat{\vartheta}(t-1)], \text{ where } \hat{\vartheta}(0) = \frac{1_{n_d}}{p_0}. \tag{33}$$

Substituting Eqs. (26)-(27) in the above equations give

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\phi(t)}{1+\|\phi(t)\|^2}[y(t) - \psi^T(t)\vartheta - \phi^T(t)\hat{\theta}(t-1)], \tag{34}$$

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + \frac{\psi(t)}{1+\|\psi(t)\|^2}[y(t) - \phi^T(t)\theta - \psi^T(t)\hat{\vartheta}(t-1)]. \tag{35}$$

And the following error is defined

$$e(t) = y(t) - \phi^T(t)\hat{\theta}(t-1) - \psi^T(t)\hat{\vartheta}(t-1). \tag{36}$$

As to enhance the performance and decrease the algorithm's sensitivity to noise, the length of data window q is used. Two step-sizes, $\mu_\theta(t)$ and $\mu_\vartheta(t)$, are used which have the following amounts

$$\mu_\theta(t) = \frac{1}{1+\|\phi\|^2}, \quad \mu_\vartheta(t) = \frac{1}{1+\|\psi\|^2}.$$

and two fictitious variables are defined as

$$r_\theta(t) := \|\phi(t)\|^2 + \|\phi(t-1)\|^2 + \cdots + \|\phi(t-q)\|^2 = r_\theta(t-1) + \|\phi(t)\|^2 + \|\phi(t-q)\|^2 \in \mathbb{R}.$$

$$r_{\vartheta}(t) := \|\psi(t)\|^2 + \|\psi(t-1)\|^2 + \cdots + \|\psi(t-q)\|^2 = \; r_{\vartheta}(t-1) + \|\psi(t)\|^2 + \|\psi(t-q)\|^2 \in \mathbb{R}.$$

Using the above definitions, μ_θ(t) and $\mu_\vartheta(t)$, can be re-defined as

$$\mu_\theta = \frac{1}{r_\theta},\ \mu_\vartheta = \frac{1}{r_\vartheta}.$$

Finally, the 2S-GPA can be made brief as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\phi(t)}{r_\phi(t)} e(t), \tag{37}$$

$$e(t) = y(t) - \phi^T(t)\hat{\theta}(t-1) - \psi^T(t)\hat{\vartheta}(t-1), \tag{38}$$

$$r_\theta(t) = r_\theta(t-1) + \|\phi(t)\|^2 + \|\phi(t-q)\|^2, \tag{39}$$

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + \frac{\psi(t)}{r_\psi(t)} e(t), \tag{40}$$

$$r_\vartheta(t) = r_\vartheta(t-1) + \|\psi(t)\|^2 + \|\psi(t-q)\|^2, \tag{41}$$

$$\phi(t) = [-y(t-1), -y(t-2), \ldots, -y(t-n_a), u(t-1), u(t-2), \ldots, u(t-n_b)]^T, \tag{42}$$

$$\psi(t) = [v(t-1), v(t-2), \ldots, v(t-n_d)]^T, \tag{43}$$

$$\hat{\theta}(t) = [\hat{a}_1(t), \hat{a}_2(t), \ldots, \hat{a}_n(t), \hat{b}_1(t), \hat{b}_2(t), \ldots, \hat{b}_n(t)]^T, \tag{44}$$

$$\hat{\vartheta}(t) = [\hat{d}_1(t), \hat{d}_2(t), \ldots, \hat{d}_n(t)]^T. \tag{45}$$

So, the identification procedure of the proposed algorithm is presented in Algorithm 2 and the flowchart is provided in Figure 3.

Algorithm 2 2S-GP algorithm.

```
t ← 1
θ̂(0) ← 1_n / p_0
ϑ̂(0) ← 1_n / p_0
r_θ(0) ← 1
r_ϑ(0) ← 1
p_0 ← 10^6
ε ← small positive value
u(t) ← collect input data
y(t) ← collect output data
Construct φ(t) and ψ(t) utilizing equations (42-43)
while ‖θ(t) − θ(t−1)‖ + ‖ϑ(t) − ϑ(t−1)‖ > ε do
    Compute e(t), r_θ(t) and r_ϑ(t) using (38-39) and (41)
    Update θ(t) and ϑ(t) according to (37) and (40)
    t ← t + 1
end while
```

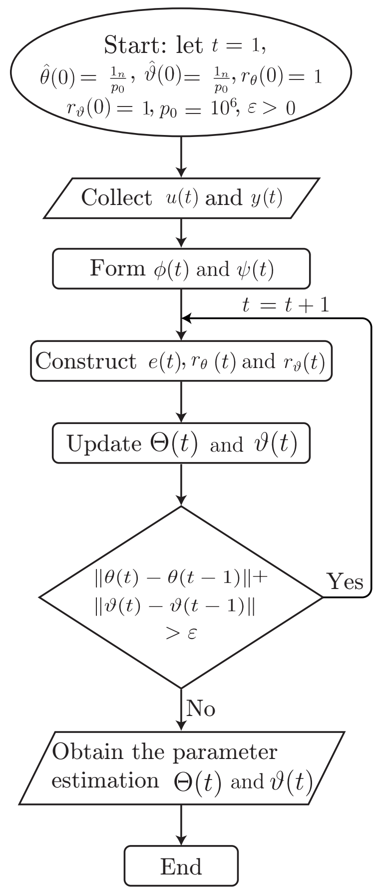


Figure 3: 2S-GPA flowchart

3 Tumor model

A tumor model described in [4] is introduced in this section. $I(t)$ indicates the immune cells number, $T(t)$ the tumor cells number, and $N(t)$ the normal or host cells number, all at time t. So we have the three equations of ordinary differential

$$\begin{aligned}
\dot{N}(t) &= r_2 N(1 - b_2 N) - c_4 T N, \\
\dot{T}(t) &= r_1 T(1 - b_1 T) - c_2 I T - c_3 T N, \\
\dot{I}(t) &= s + \frac{\rho I T}{\alpha + T} - c_1 I T - d_1 I.
\end{aligned} \tag{46}$$

In principle, the goal is for the tumor-free equilibrium to be stable, so that there is a possibility of moving the system's state toward the tumor-free point. Linearizing around this equilibrium point leads to the following system

$$\begin{bmatrix} \dot{N}(t) \\ \dot{T}(t) \\ \dot{I}(t) \end{bmatrix} = \begin{bmatrix} r_2 - 2r_2 b_2 & -c_4 & 0 \\ 0 & r_1 - \frac{c_2 f}{d_1} - c_3 & 0 \\ 0 & \frac{\rho f}{d_1 \alpha} - \frac{c_1 f}{d_1} & -d_1 \end{bmatrix} \begin{bmatrix} N(t) \\ T(t) \\ I(t) \end{bmatrix}. \tag{47}$$

Table 1: System values and parameters

parameters	values	parameters	values
b_2	1	α	0.3
c_1	1	c_2	0.5
c_3	1	c_4	1
d_1	0.2	ρ	0.01
r_1	1.5	r_2	1
f	0.33		

As a study case for identification in the next section, the accurate parameters which are used, are brought up in Table 1 considering the following state-space equations

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{48}$$
$$y(t) = C(t)x(t) + D(t)u(t). \tag{49}$$

here $x(t) = [N(t), T(t), I(t)]^T \in \mathbb{R}^3$, $u(t)$, indicates the amount of drug at the tumor place at time t . The state matrix, $A(s)$, for equation (47), in continuous time, is modeled as

$$A(s) = \begin{bmatrix} s+1 & 1 & 0 \\ 0 & s+0.33 & 0 \\ 0 & 1.595 & s+0.2 \end{bmatrix}.$$

Therefor, $G(s)$ equals

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{-100}{(100s+33)(s+1)} & 0 \\ 0 & \frac{100}{100s+33} & 0 \\ 0 & \frac{-1595}{2(5s+1)(100s+33)} & \frac{5}{5s+1} \end{bmatrix}.$$

Since overcoming the tumor problem is an important matter, having a polynomial form of the transfer function G(s) plays a vital role in controlling the procedure [10], as polynomial models can estimate all the variables of the model with great precision.

4 Simulations

In this section, we aim to identify $G_{12}(s) = \frac{-100}{(100s+33)(s+1)}$ and $G_{32}(s) = \frac{-1595}{2(5s+1)(100s+33)}$. In the simulations, $u(t)$ is assumed as a sequence of signal of uncorrelated persistent excitation with unit variance and zero mean , also $v(t)$ is a sequence of white noise with variance σ^2 and zero mean. Also, the error of parameter estimation (δ) is reported as $\delta = \frac{\|\hat{\Theta}(t)-\Theta(t)\|}{\|\Theta(t)\|}$. Note that the lower the delta, the better the identification performance. Furthermore $t = N$ represents the number of data in the procedure of the parameter estimation. In the following two sections, two transfer functions are tried to be identified, and the performances are reported.

Estimation of $G_{12}(s)$

From the perspective of CARMA, $G_{12}(s)$ can be modeled as follows

Table 2: Estimation results for $q = 100$ and $\sigma^2 = (1.00)^2$.

Approach	$t = N$	a_1	a_2	b_1	b_2	d_1	d_2	$\delta(\%)$
GPA	1000	-0.9080	-0.0908	0.2690	-0.0609	-0.2804	0.4681	49.0273
	2000	-0.9248	-0.0638	0.4983	0.0696	-0.7048	0.0367	41.4382
	3000	-1.1342	0.1362	0.2560	0.0246	-0.3186	0.4073	37.9335
2S-GPA	1000	-1.0500	0.0530	0.6183	-0.1093	-0.2400	-0.5425	47.7595
	2000	-1.0360	0.0375	0.5980	-0.0445	-0.1932	0.1980	41.0527
	3000	-0.9583	-0.0373	0.5109	-1.1668	-0.2184	0.0702	46.4776
True Value		-1.9080	0.9086	0.7296	-0.7296	-1.0600	0.2807	

Table 3: Estimation results for $q = 100$ and $\sigma^2 = (2.00)^2$.

Approach	$t = N$	a_1	a_2	b_1	b_2	d_1	d_2	$\delta(\%)$
GPA	1000	-0.9442	-0.0550	0.7531	0.4499	-0.6365	0.6253	52.6830
	2000	-2.0227	1.0278	-0.5802	0.3416	-0.9448	0.6348	44.9876
	3000	-1.1218	0.1244	1.2679	0.2656	-0.5780	0.5487	41.8460
2S-GPA	1000	-1.0778	0.0813	0.4067	-0.1362	0.1653	0.0270	50.4326
	2000	-1.2889	0.2911	0.2515	0.2047	0.1217	0.0424	49.3000
	3000	-1.0881	0.0940	0.4516	0.0141	-0.5511	-0.6656	46.3470
True Value		-1.9080	0.9086	0.7296	-0.7296	-1.0600	0.2807	

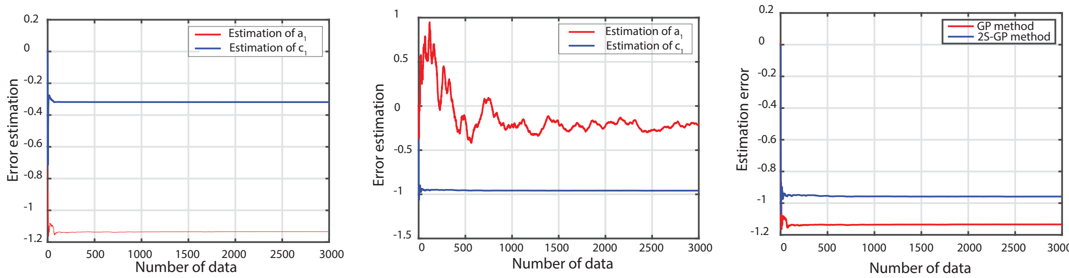
$$A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} = 1 - 1.9080q^{-1} + 0.9086q^{-2},$$
$$B(q) = b_1 q^{-1} + b_2 q^{-2} = 0.7296q^{-1} - 0.7296q^{-2},$$
$$D(q) = 1 + d_1 q^{-1} + d_2 q^{-2} = 1 - 1.0600q^{-1} + 0.2807q^{-2}.$$

In the above equations, the true values of parameters can be seen, e.g., the true value of a_1 is -1.9080. The performance of the introduced algorithms are reported through six different cases in Table (2-3), which covers two different variances, σ, with different number of data, L.

4.1 Estimation of $G_{32}(s)$

Consider $G_{32}(s)$, which is introduced before. The CARMA system related to this transfer function contains the following set of modelling equations

$$A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} = 1 - 1.8820q^{-1} + 0.8851q^{-2},$$
$$B(q) = b_1 q^{-1} + b_2 q^{-2} = -7.9590q^{-1} + 7.9590q^{-2},$$
$$D(q) = 1 + d_1 q^{-1} + d_2 q^{-2} = 1 - 1.1620q^{-1} + 0.3821q^{-2}.$$



(a) **(b)** **(c)**

Figure 4: **(a)** Estimation of a_1 and c_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$ with GP algorithm. **(b)** Estimation of a_1 and c_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$ with 2S-GP algorithm. **(c)** Estimation of a_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$.

Table 4: Estimation results for $q = 100$ and $\sigma^2 = (1.00)^2$.

Approach	$t = N$	a_1	a_2	b_1	b_2	d_1	d_2	$\delta(\%)$
GPA	1000	-0.8823	-0.0317	-6.6629	-0.1044	2.1965	0.0088	60.3393
	2000	-0.8996	0.0004	-5.2713	0.1614	-2.2398	0.1304	53.5796
	3000	-0.9199	0.0082	-5.4179	0.1293	-2.4079	0.0805	53.6542
2S-GPA	1000	-0.8495	-0.0500	-4.9606	0.1672	-2.9524	2.7582	60.7443
	2000	-0.9021	-0.0081	-5.7461	0.1347	0.2839	0.2978	52.8040
	3000	-0.8709	-0.0380	-5.8848	0.1548	-0.6321	-0.6473	51.6348
True Value		-1.8820	0.8851	-7.9590	7.9590	-1.6120	0.3821	

Table 5: Estimation results for $q = 100$ and $\sigma^2 = (2.00)^2$.

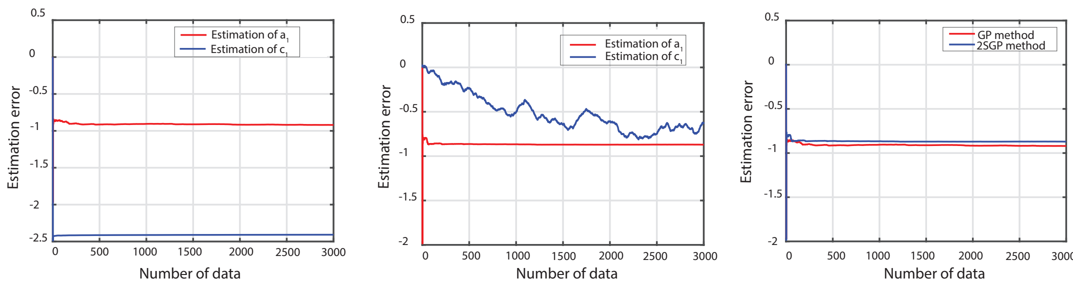
Approach	$t = N$	a_1	a_2	b_1	b_2	d_1	d_2	$\delta(\%)$
GPA	1000	-1.2686	0.3488	-2.9392	0.2850	-0.4752	-1.0802	65.9273
	2000	-0.9256	-0.0118	-4.2711	-0.0020	-2.7460	0.0098	61.3865
	3000	-0.9245	-0.0143	-5.7686	-0.0933	0.3549	0.0059	55.6963
2S-GPA	1000	-0.8493	-0.0716	-3.7840	0.0069	-0.0412	-0.0491	63.4575
	2000	-1.0680	0.1261	-2.6923	0.9499	-0.3062	-0.3065	59.8514
	3000	-0.9859	0.0457	-5.6405	-0.1127	0.3809	0.3699	56.1575
True Value		-1.8820	0.8851	-7.9590	7.9590	-1.6120	0.3821	

The results of the identification procedure with the introduced methods, which are GPA and 2S-GPA, are reported in Tables 2-5.

From Tables 2-5 and Figs. 4-5 these deductions are derived

1. The system identification errors of the GPA and 2S-GPA approaches decrease as the data length increases.

2. 2S-GPA method compared to GPA method, produces less error and therefore is more effective at



(a) **(b)** **(c)**

Figure 5: **(a)** Estimation of a_1 and c_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$ with GP algorithm. **(b)** Estimation of a_1 and c_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$ with 2S-GP algorithm. **(c)** Estimation of a_1 for CARMA system with variance $\sigma^2 = 1.00^2$, number of data $L = 3000$ and $q = 100$.

estimating parameters.

3. As the noise to ratio signal rises, both introduced algorithms produce more considerable amount of error.

4. From depicted figures, it is perceived that both introduced algorithms converge to the same point and have competent convergence rate.

In this contribution, all advantages and disadvantages of proposed algorithm were discussed. Clearly the advantages outweigh the disadvantages, and reader would have all necessary information to compare given approaches and design a suitable controller for tumor model or any desired system.

5 Conclusion

In this research, two system identification approaches for estimating parameters of a CARMA system were presented. It is shown that generalized projection algorithm and two-stage generalize projection algorithm both can converge to the actual values of the system at a fast rate, and both produce an insignificant amount of error, especially for a large number of data. However, the two-stage method is depicted to be more efficient at error reduction of tumor model at its tumor-free equilibrium. Furthermore, the work done in this paper could be later used in order to control complicated systems such as the tumor model with the existence of significant noise in the system.

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