Journal of Mathematical Modeling
Vol. 12, No. 4, 2024, pp. 769–779. Research Article

JMM

Stabilization by delay feedback control for highly nonlinear HSDDEs driven by Lévy noise

Zhihao Geng*

School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo 454003, China
Email(s): hpugzh@163.com

Abstract. This research aims to investigate the stabilization of highly nonlinear hybrid stochastic differential delay equations (HSDDEs) with Lévy noise by delay feedback control. The coefficients of these systems satisfy a more general polynomial growth condition instead of classical linear growth condition. Precisely, an appropriate Lyapunov functional is constructed to analyze the stabilization of such systems in the sense of H_∞-stability and asymptotic stability. The theoretical analysis indicates that the delay can affect the stability of highly nonlinear hybrid stochastic systems.

Keywords: Stabilization, Lévy noise, stochastic differential delay equations, delay feedback control.
AMS Subject Classification 2010: 93B52, 93E03.

1 Introduction

In mathematics and engineering, the dynamics of many stochastic differential systems are often influenced by both their current and past states (see e.g., Mao et al. [23], Bahar et.al. [3]). Stochastic differential delay equations (SDDEs) are commonly used to model these systems with delays. However, many real systems may undergo abrupt changes in their structure and parameters. Therefore, HSDDEs are used to model these practical systems (see e.g., [4, 8, 22, 28]). The HSDDEs can be described by

$$\mathrm{d}x(t) = \sigma(x(t), x(t-\omega(t)), \xi(t), t)\mathrm{d}t + \varphi(x(t), x(t-\omega(t)), \xi(t), t)\mathrm{d}B(t), \tag{1}$$

where $x(t)$ and $\xi(t)$ are often referred to as the state and mode respectively, and $\xi(t)$ is a Markov chain which takes its values in $S = \{1, 2, \ldots, N\}$. There is already a vast amount of literature on the stabilization of HSDDEs (see e.g., [5,7,20,26,30]). It is well known that the feedback control based on the continuous-time state observations (or based on the discrete-time state observations) can stabilize an unstable system or destabilize a stable system. Using the continuous-time feedback control, most researchers study the

*Corresponding author
Received: 15 August 2024 / Revised: 25 August 2024 / Accepted: 28 August 2024
DOI: 10.22124/jmm.2024.28174.2486

© 2024 University of Guilan

stabilization and destabilization of HSDDEs systems (see e.g., [14, 19, 29]). For example, Lu et al. [19] studied the asymptotically stability of unstable hybrid stochastic differential equation system via feedback control.

In fact, many practical systems are influenced not only by time delays but also by the perturbation of random factors. In order to model more realistic systems, many literatures have incorporated Lévy noise into HSDDEs (see e.g., [10, 13, 15, 18, 31]). The HSDDEs with Lévy noise can be described by

$$\begin{aligned} \mathrm{d}x(t) =&\sigma(x(t),x(t-\omega(t)),\xi(t),t)\mathrm{d}t+\varphi(x(t),x(t-\omega(t)),\xi(t),t)\mathrm{d}B(t)\\ &+\int_L \phi(x(t),x(t-\omega(t)),\xi(t),t,l)\tilde{N}(\mathrm{d}t,\mathrm{d}l), \end{aligned} \tag{2}$$

where $N(\cdot,\cdot)$ is a Poisson random measure on $[0,+\infty)\times R^n$ with compensator $\tilde{N}(\mathrm{d}t,\mathrm{d}l)=N(\mathrm{d}t,\mathrm{d}l)-\lambda(\mathrm{d}l)\mathrm{d}t$, where λ is the intensity measure. An important issue in the investigation of HSDDEs with Lévy noise is the stochastic stabilization. Many studies with Lévy noise discussed the stability and stabilization under the influence of linear conditions (see e.g., [2, 6, 11, 16, 17, 21, 27]). Li et al. [16] showed the exponential stability of the controlled hybrid stochastic differential equations with Lévy noise. However, the stabilization of highly nonlinear HSDDEs driven by Lévy noise has not been fully investigated, which motivates the present work. This paper is different from the aforementioned literature as more techniques are needed to overcome the difficulties of both Lévy noise and highly nonlinear conditions. Therefore, the study of the stabilization in the present work is more complex and difficult.

The organization of this paper is described as follows. The foundational concepts regarding HSDDEs with Lévy noise and some conditions are provided in Section 2. We consider the influence of delay on the stability of HSDDEs system in Section 3. Finally, some conclusions are presented in Section 4.

2 Problem formulation and preliminaries

Let $(\Omega,\mathfrak{F},\{\mathfrak{F}_t\}_{t\geq 0},\mathbb{P})$ be a complete probability space, and $B(t)$ be an m-dimensional Brownian motion. Define $Q=(q_{ij})_{N\times N}$ as the generator of Markov chain $\xi(t)$. Let η be a positive constant and $\bar{\omega}$ be a value in $[0,1)$. Define $\omega(\cdot)$ from R_+ to $[0,\eta]$ as a differentiable function with the condition $\dot{\omega}(t):=\mathrm{d}\omega(t)/\mathrm{d}t\leq\bar{\omega}$ for each $t\geq 0$. If W is a matrix, then W^{T} represents its transpose. The trace norm of W is $|W|=\sqrt{\mathrm{trace}(W^{\mathrm{T}}W)}$. Let $C([-\eta,0];R^n)$ denote the family of continuous functions μ from $[-\eta,0]$ to R^n. The norm of μ is $\|\mu\|=\sup_{-\eta<\theta<0}|\mu(\theta)|$.

Consider an unstable HSDDEs system with Lévy noise in the form of (2). Our aim is to design a delay feedback control $u(x(t-\omega(t)),\xi(t),t)$ for the controlled system

$$\begin{aligned} \mathrm{d}x(t)=&\ [\sigma(x(t),x(t-\omega(t)),\xi(t),t)+u(x(t-\omega(t)),\xi(t),t)]\mathrm{d}t+\varphi(x(t),x(t-\omega(t)),\xi(t),t)\mathrm{d}B(t)\\ &+\int_L \phi(x(t),x(t-\omega(t)),\xi(t),t,l)\tilde{N}(\mathrm{d}t,\mathrm{d}l) \end{aligned} \tag{3}$$

to be stable. We give the initial value

$$x_0=\varepsilon=\{x(t):-\eta\leq t\leq 0\}\in C([-\eta,0];R^n) \text{ and } \xi(0)=\xi_0\in S, \tag{4}$$

where $\sigma: R^n\times R^n\times S\times R_+\to R^n$, $u: R^n\times S\times R_+\to R^n$, $\varphi: R^n\times R^n\times S\times R_+\to R^{n\times m}$ and $\phi: R^n\times R^n\times S\times R_+\times R^n\to R^n$. Assume that $\xi(t)$, $B(t)$ and $N(t,l)$ are independent of each other. To guarantee the existence and uniqueness of the global solution of (3), the following assumptions are necessary.

Assumption 1. *Suppose that $H_c > 0$ for each $c > 0$ and*

$$\begin{aligned}&|\sigma(x_1,y_1,i,t)-\sigma(x_2,y_2,i,t)|\vee|\varphi(x_1,y_1,i,t)-\varphi(x_2,y_2,i,t)|\\&\vee\int_L|\phi(x_1,y_1,i,t,l)-\phi(x_2,y_2,i,t,l)|\lambda(\mathrm{d}l)\le H_c(|x_1-x_2|+|y_1-y_2|)\end{aligned}\tag{5}$$

for $|x_1|\vee|y_1|\vee|x_2|\vee|y_2|\le c$.

Assumption 2. *Suppose that there are numbers $m_1 > 1$, $m_2 > 1$, $m_3 > 1$ and $H > 0$ such that*

$$\begin{aligned}&|\sigma(x,y,i,t)|\le H(1+|x|^{m_1}+|y|^{m_1}),\\&|\varphi(x,y,i,t)|\le H(1+|x|^{m_2}+|y|^{m_2}),\\&|\phi(x,y,i,t,l)|\le H(1+|x|^{m_3}+|y|^{m_3}).\end{aligned}\tag{6}$$

We will refer to Assumption 2 as the polynomial growth condition. Moreover, a global Lipschitz condition on the controller function u is required.

Assumption 3. *Suppose that there exists a positive constant k_5 such that*

$$|u(x,i,t)-u(y,i,t)|\le k_5|x-y|,\tag{7}$$

and $u(0,i,t)=0$.

Let us now recall the following definitions of H_∞-stability and asymptotic stability.

Definition 1. (Mao et al. [24]) The trivial solution of (3) is said to be H_∞-stable if for any $x_0\in\mathbb{R}^n$,

$$\int_0^\infty \mathbb{E}|x(t)|^p\mathrm{d}t<0 \quad a.s..$$

It is said to be asymptotically stable if for any $x_0\in\mathbb{R}^n$,

$$\lim_{t\to\infty}\mathbb{E}|x(t)|^p=0 \quad a.s..$$

Then, from Rhaima et al. [25] and Li et al. [12], we can find that hybrid system (3) has a unique global solution. Let $C^{2,1}(R^n\times S\times R_+;R_+)$ be the family of non-negative function $V(x,i,t)$ which is continuously twice differentiable in x and once in t, define LV as in [16] by

$$\begin{aligned}LV(x,i,t)=&V_t(x,i,t)+V_x(x,i,t)[\sigma(x,y,i,t)+u(y,i,t)]\\&+\int_L\left[V(x+\phi(x,y,i,t,l),i,t)-V(x,i,t)-V_x(x,i,t)\phi(x,y,i,t,l)\right]\lambda(\mathrm{d}l)\\&+\frac{1}{2}\,\mathrm{trace}\left[\varphi^T(x,y,i,t)V_{xx}(x,i,t)\varphi(x,y,i,t)\right]+\sum_{j=1}^N\gamma_{ij}V(x,j,t).\end{aligned}$$

3 Stochastic stabilization by Lévy noise

To achieve stabilization for the hybrid system (3), we intend to employ a Lyapunov function on the segment processes $\bar{x}_t = \{x(t+u) : -2\eta \le u \le 0\}$ and $\bar{\xi}_t = \{\xi(t+u) : -2\eta \le u \le 0\}$ for $t \ge 0$. To guarantee the proper definition of $\bar{x}_t$ and $\bar{\xi}_t$ for $0 \le t < 2\eta$, we set $x(u) = \varepsilon(-\eta)$ for $u \in [-2\eta, -\eta)$ and $\xi(u) = \xi_0$ for $u \in [-2\eta, 0)$. Define a Lyapunov functional as

$$V(\bar{x}_t, \bar{\xi}_t, t) = M(x(t), \xi(t), t) + \tau \int_{-\eta}^{0} \int_{t+u}^{t} \Big[\eta |\sigma(x(s), x(s-\omega(s)), \xi(s), s) + u(x(s-\omega(s)), \xi(s), s)|^2 + |\varphi(x(s), x(s-\omega(s)), \xi(s), s)|^2 + \int_L |\phi(x(s), x(s-\omega(s)), \xi(s), s, l)|^2 \lambda(\mathrm{d}l) \Big] \mathrm{d}s\mathrm{d}u, \tag{8}$$

where $\tau > 0$ and $M \in C^{2,1}(R^n \times S \times R_+; R_+)$. In our discussion of stability for the controlled system (3), we need to make the following assumption.

Assumption 4. *For the functions $M \in C^{2,1}(R^n \times S \times R_+; R_+)$, M_1, $M_2 \in C(R^n \times [-\eta, \infty); R_+)$ and constants ρ_m, $k_m > 0$ $(m = 1,2,3,4)$, assume that*

$$\begin{aligned} &\mathbb{L}M(x,y,i,t) + k_1 |M_x(x,i,t)|^2 + k_2 |\sigma(x,y,i,t) + u(y,i,t)|^2 + k_3 |\varphi(x,y,i,t)|^2 \\ &\quad + k_4 \int_L |\phi(x,y,i,t,l)|^2 \lambda(\mathrm{d}l) \\ &\quad \le -\rho_1 M_1(x,t) + \rho_2 M_1(y, t-\omega(t)) - \rho_3 M_2(x,t) + \rho_4 M_2(y, t-\omega(t)), \end{aligned} \tag{9}$$

where

$$\rho_2 < \rho_1(1-\bar{\omega}), \quad \rho_4 \le \rho_3(1-\bar{\omega}), \tag{10}$$

and $\mathbb{L}M(x,y,i,t)$ is indicated by condition (17) below.

Theorem 1. *Under Assumptions 1, 2, 3 and 4, suppose that*

$$c|x|^p \le M_1(x,t), \tag{11}$$

and

$$\eta \le \frac{2\sqrt{3k_1k_2}}{3k_5} \wedge \frac{4k_1k_3}{3k_5^2} \wedge \frac{4k_1k_4}{3k_5^2} \tag{12}$$

for c, $p > 0$. Then

$$\int_0^\infty \mathbb{E}|x(t)|^p \mathrm{d}t < \infty. \tag{13}$$

Consequently, the controlled system (3) is H_∞-stable.

Proof. Fix the initial condition (4). Let $h_0 > 0$ be a big enough positive integer such that $\|x_0\| = \|\varepsilon\| < h_0$. For any integer $h \ge h_0$, the stopping time δ_h is defined as $\delta_h = \inf\{t \ge 0 : |x(t)| \ge h\}$. Noting that the stopping time δ_h is increasing as $h \to \infty$. Therefore, we conclude that $\lim_{h\to\infty} \delta_h = \infty$. Applying the generalised Itô formula (see e.g., Mao and Yuan [24], D. Applebaum [1]) to $V(\bar{x}_t, \bar{\xi}_t, t)$ yields,

$$\mathrm{d}V(\bar{x}_t, \bar{\xi}_t, t) = LV(\bar{x}_t, \bar{\xi}_t, t)\mathrm{d}t + P(t), \tag{14}$$

where

$$
\begin{aligned}
P(t) = {} & M_x(x(t),\xi(t),t)\varphi(x(t),x(t-\omega(t)),\xi(t),t)\mathrm{d}B(t) \\
& + \int_L \big(M(x(t)+\phi(x(t),x(t-\omega(t)),\xi(t),t,l),\xi(t),t) - M(x(t),\xi(t),t)\big)\tilde{N}(\mathrm{d}t,\mathrm{d}l) \\
& + \int_R \big(M(x(t),\xi_0 + c(\xi(t),z),t) - M(x(t),\xi(t),t)\big)\mu(\mathrm{d}t,\mathrm{d}z),
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
LV(\bar{x}_t,\bar{\xi}_t,t) = {} & M_x(x(t),\xi(t),t)\times[u(x(t-\omega(t)),\xi(t),t) - u(x(t),\xi(t),t)] \\
& + \mathbb{L}M(x(t),x(t-\omega(t)),\xi(t),t) \\
& + \tau\eta\Big[\eta|\sigma(x(t),x(t-\omega(t)),\xi(t),t) + u(x(t-\omega(t)),\xi(t),t)|^2 \\
& + |\varphi(x(t),x(t-\omega(t)),\xi(t),t)|^2 + \int_L |\phi(x(t),x(t-\omega(t)),\xi(t),t,l)|^2\lambda(\mathrm{d}l)\Big] \\
& - \tau\int_{t-\eta}^{t}\Big[\eta|\sigma(x(s),x(s-\omega(s)),\xi(s),s) + u(x(s-\omega(s)),\xi(s),s)|^2 \\
& + |\varphi(x(s),x(s-\omega(s)),\xi(s),s)|^2 + \int_L |\phi(x(s),x(s-\omega(s)),\xi(s),s,l)|^2\lambda(\mathrm{d}l)\Big]\mathrm{d}s,
\end{aligned} \tag{16}
$$

in which $\mathbb{L}M$ is defined by

$$
\begin{aligned}
\mathbb{L}M(x,y,i,t) = {} & M_t(x,i,t) + M_x(x,i,t)\sigma(x,y,i,t) + M_x(x,i,t)u(x,i,t) \\
& + \int_L \big[M(x+\phi(x,y,i,t,l),i,t) - M(x,i,t) - M_x(x,i,t)\phi(x,y,i,t,l)\big]\lambda(\mathrm{d}l) \\
& + \frac{1}{2}\,\mathrm{trace}\big[\varphi^T(x,y,i,t)M_{xx}(x,i,t)\varphi(x,y,i,t)\big] + \sum_{j=1}^{N}\gamma_{ij}M(x,j,t).
\end{aligned} \tag{17}
$$

From condition (14), we can obtain that

$$
\mathbb{E}V\left(\bar{x}_{t\wedge\delta_h},\bar{\xi}_{t\wedge\delta_h},t\wedge\delta_h\right) = V\left(\bar{x}_0,\bar{\xi}_0,0\right) + \mathbb{E}\int_0^{t\wedge\delta_h} LV\left(\bar{x}_r,\bar{\xi}_r,r\right)\mathrm{d}r. \tag{18}
$$

Let $\tau = 3k_5^2/4k_1$, from Assumptions 3, 4 and condition (12), we have

$$
\begin{aligned}
& LV\left(\bar{x}_r,\bar{\xi}_r,r\right) \\
& \le -\rho_1 M_1(x(r),r) + \rho_2 M_1(x(r-\omega(r)),r-\omega(r)) - \rho_3 M_2(x(r),r) + \rho_4 M_2(x(r-\omega(r)),r-\omega(r)) \\
& \quad + \frac{k_5^2}{4k_1}|x(r)-x(r-\omega(r))|^2 - \frac{3k_5^2}{4k_1}\int_{r-\eta}^{r}\big[\eta|\sigma(x(s),x(s-\omega(s)),\xi(s),s) + u(x(s-\omega(s)),\xi(s),s)|^2 \\
& \quad + |\varphi(x(s),x(s-\omega(s)),\xi(s),s)|^2 + \int_L |\phi(x(s),x(s-\omega(s)),\xi(s),s,l)|^2\lambda(\mathrm{d}l)\big]\mathrm{d}s.
\end{aligned} \tag{19}
$$

Note that

$$
\int_0^{t\wedge\delta_h} M_1(x(r-\omega(r)),r-\omega(r))\mathrm{d}r \le \int_{-\eta}^{t\wedge\delta_h} M_1(x(r),r)\mathrm{d}r.
$$

Substituting (19) into (18) yields that

$$
\begin{aligned}
&\mathbb{E}V(\bar{x}_{t\wedge\delta_h},\bar{\xi}_{t\wedge\delta_h},t\wedge\delta_h)\\
&\quad\le V\left(\bar{x}_0,\bar{\xi}_0,0\right)+\frac{\rho_2}{1-\bar{\omega}}\int_{-\eta}^{0}M_1(\varepsilon(s),s)\mathrm{d}s-\bar{\rho}\mathbb{E}\int_0^{t\wedge\delta_h}M_1(x(r),r)\mathrm{d}r\\
&\qquad+\frac{\rho_4}{1-\bar{\omega}}\int_{-\eta}^{0}M_2(\varepsilon(s),s)\mathrm{d}s+\frac{k_5^2}{4k_1}\mathbb{E}\int_0^{t\wedge\delta_h}|x(r)-x(r-\omega(r))|^2\mathrm{d}r\\
&\qquad-\frac{3k_5^2}{4k_1}\mathbb{E}\int_0^{t\wedge\delta_h}\Big(\int_{r-\eta}^{r}\big[\eta|\sigma(x(s),x(s-\omega(s)),\xi(s),s)+u(x(s-\omega(s)),\xi(s),s)|^2\\
&\qquad+|\varphi(x(s),x(s-\omega(s)),\xi(s),s)|^2+\int_L|\phi(x(s),x(s-\omega(s)),\xi(s),s,l)|^2\lambda(\mathrm{d}l)\big]\mathrm{d}s\Big)\mathrm{d}r,
\end{aligned}
\tag{20}
$$

where $\bar{\rho}=\rho_1-\rho_2/(1-\bar{\omega})>0$ from condition (10). Applying the Fatou lemma (see [9]) and letting $h\to\infty$ in (20), we can get that

$$
\bar{\rho}\mathbb{E}\int_0^t M_1(x(r),r)\mathrm{d}r\le D_1+I_1-I_2,
\tag{21}
$$

where

$$
\begin{aligned}
D_1&=V\left(\bar{x}_0,\bar{\xi}_0,0\right)+\frac{\rho_2}{1-\bar{\omega}}\int_{-\eta}^{0}M_1(\varepsilon(s),s)\mathrm{d}s+\frac{\rho_4}{1-\bar{\omega}}\int_{-\eta}^{0}M_2(\varepsilon(s),s)\mathrm{d}s,\\
I_1&=\frac{k_5^2}{4k_1}\mathbb{E}\int_0^t|x(r)-x(r-\omega(r))|^2\mathrm{d}r,\\
I_2&=\frac{3k_5^2}{4k_1}\mathbb{E}\int_0^t\Big(\int_{r-\eta}^{r}\big[\eta|\sigma(x(s),x(s-\omega(s)),\xi(s),s)+u(x(s-\omega(s)),\xi(s),s)|^2\\
&\quad+|\varphi(x(s),x(s-\omega(s)),\xi(s),s)|^2+\int_L|\phi(x(s),x(s-\omega(s)),\xi(s),s,l)|^2\lambda(\mathrm{d}l)\big]\mathrm{d}s\Big)\mathrm{d}r.
\end{aligned}
$$

From the Fubini theorem, we have

$$
I_1=\frac{k_5^2}{4k_1}\int_0^t\mathbb{E}|x(r)-x(r-\omega(r))|^2\mathrm{d}r.
\tag{22}
$$

For $t\in[0,\eta]$, we get that

$$
I_1\le\frac{\eta k_5^2}{k_1}\left(\sup_{-\eta\le s\le\eta}\mathbb{E}|x(s)|^2\right)=:D_2.
\tag{23}
$$

Therefore we have

$$
I_1\le D_2+\frac{k_5^2}{4k_1}\int_\eta^t\mathbb{E}|x(r)-x(r-\omega(r))|^2\,\mathrm{d}r
\tag{24}
$$

for $t > \eta$. Moreover, according to Itô isometry and Eq. (3), we obtain that

$$
\begin{aligned}
&\mathbb{E}|x(r)-x(r-\omega(r))|^2\\
=&\mathbb{E}\Big|\int_{r-\eta}^{r}\sigma(x(s),x(s-\omega(s)),\xi(s),s)+u(x(s-\omega(s)),\xi(s),s)\mathrm{d}s\\
&+\int_{r-\eta}^{r}\varphi(x(s),x(s-\omega(s)),\xi(s),s)\mathrm{d}B(s)+\int_{r-\eta}^{r}\int_L\phi(x(s),x(s-\omega(s)),\xi(s),s,l)\tilde{N}(\mathrm{d}s,\mathrm{d}l)\Big|^2\\
\le&3\eta\mathbb{E}\int_{r-\eta}^{r}|\sigma(x(s),x(s-\omega(s)),\xi(s),s)+u(x(s-\omega(s)),\xi(s),s)|^2\mathrm{d}s\\
&+3\mathbb{E}\Big|\int_{r-\eta}^{r}\varphi(x(s),x(s-\omega(s)),\xi(s),s)\mathrm{d}B(s)\Big|^2+3\mathbb{E}\Big|\int_{r-\eta}^{r}\int_L\phi(x(s),x(s-\omega(s)),\xi(s),s,l)\tilde{N}(\mathrm{d}s,\mathrm{d}l)\Big|^2\\
\le&3\eta\mathbb{E}\int_{r-\eta}^{r}|\sigma(x(s),x(s-\omega(s)),\xi(s),s)+u(x(s-\omega(s)),\xi(s),s)|^2\mathrm{d}s\\
&+3\mathbb{E}\int_{r-\eta}^{r}\Big|\varphi(x(s),x(s-\omega(s)),\xi(s),s)\Big|^2\mathrm{d}s+3\mathbb{E}\int_{r-\eta}^{r}\int_L\Big|\phi(x(s),x(s-\omega(s)),\xi(s),s,l)\Big|^2\lambda(\mathrm{d}l)\mathrm{d}s
\end{aligned}
\tag{25}
$$

for $r \ge \eta$. So it is easy to see that $I_1 \le D_2 + I_2$ for $t > \eta$. That is,

$$I_1 \le D_2 + I_2, \quad t \ge 0. \tag{26}$$

Substituting (26) into (21), we have that

$$\bar{\rho}\mathbb{E}\int_0^t M_1(x(r),r)\mathrm{d}r \le D_1 + D_2. \tag{27}$$

Let $t \to \infty$, and by the Fubini theorem, we obtain that

$$\int_0^\infty \mathbb{E}M_1(x(r),r)\mathrm{d}r \le (D_1+D_2)/\bar{\rho} < \infty. \tag{28}$$

Then from (11) and (28), the solution to (3) satisfies

$$\int_0^\infty \mathbb{E}|x(t)|^p\mathrm{d}t < \infty.$$

Therefore we complete the proof. □

Theorem 1 indicates that certain selections of delay feedback control $u(x(t-\omega(t)),\xi(t),t)$ are feasible for stabilizing the hybrid system (2). Then, our work will focus on asymptotic stability of the controlled system (3). To this end, we also need $\mathbb{E}|x(t)|^p$ to be uniformly continuous in t besides H_∞-stable. Let us now give an assumption.

Assumption 5. *Suppose that there exist some positive numbers k_6, k_7 and k_8 such that*

$$
\begin{aligned}
&\int_L\Big(|x(t)+\phi(x(t),x(t-\omega(t)),\xi(t),t,l)|^p\\
&\qquad -|x(t)|^p-p|x(t)|^{p-2}x^T(t)|\phi(x(t),x(t-\omega(t)),\xi(t),t,l)|\Big)\lambda(\mathrm{d}l)\\
&\le k_6+k_7|x(t)|^m+k_8|x(t-\omega(t))|^m.
\end{aligned}
\tag{29}
$$

Theorem 2. *Let all the conditions of Theorem 1 hold. If*

$$p \geq 2 \ \ and \ \ pm_3 \vee (p+m_1-1) \vee (p+2m_2-2) \leq m, \tag{30}$$

then (3) with the initial value (4) satisfies

$$\lim_{t\to\infty} \mathbb{E}|x(t)|^p = 0.$$

Namely, the controlled system (3) is asymptotically stable.

Proof. Utilizing the generalised Itô formula to $|x(t)|^p$ yields that

$$\begin{aligned}\mathbb{E}|x(t_2)|^p - \mathbb{E}|x(t_1)|^p = \ & \mathbb{E}\int_{t_1}^{t_2} \Big(p|x(t)|^{p-2}x^T(t)|\sigma(x(t),x(t-\omega(t)),\xi(t),t) + u(x(t-\omega(t)),\xi(t),t)| \\ & + \frac{p}{2}|x(t)|^{p-2}|\varphi(x(t),x(t-\omega(t)),\xi(t),t)|^2 \\ & + \frac{p(p-2)}{2}|x(t)|^{p-4}|x^T(t)\varphi(x(t),x(t-\omega(t)),\xi(t),t)|^2 \\ & + \int_L \big[|x(t)+\phi(x(t),x(t-\omega(t)),\xi(t),t,l)|^p - |x(t)|^p \\ & - p|x(t)|^{p-2}x^T(t)|\phi(x(t),x(t-\omega(t)),\xi(t),t,l)|\big]\lambda(\mathrm{d}l)\Big)\mathrm{d}t\end{aligned} \tag{31}$$

for any $0 \leq t_1 < t_2 < \infty$. By condition (6), Assumption 5 and the inequality $(a+b+c)^2 \leq 3(a^2+b^2+c^2)$, we obtain that

$$\begin{aligned}& \big|\mathbb{E}|x(t_2)|^p - \mathbb{E}|x(t_1)|^p\big| \\ & \leq \ \mathbb{E}\int_{t_1}^{t_2} \Big(p|x(t)|^{p-1}|\sigma(x(t),x(t-\omega(t)),\xi(t),t) + u(x(t-\omega(t)),\xi(t),t)| \\ & \quad + \frac{p(p-1)}{2}|x(t)|^{p-2}|\varphi(x(t),x(t-\omega(t)),\xi(t),t)|^2 \\ & \quad + k_6 + k_7|x(t)|^m + k_8|x(t-\omega(t))|^m\Big)\mathrm{d}t \\ & \leq \ \mathbb{E}\int_{t_1}^{t_2} \Big(pH|x(t)|^{p-1}[1+|x(t)|^{m_1} + |x(t-\omega(t))|^{m_1}] + pk_5|x(t)|^{p-1}|x(t-\omega(t))| \\ & \quad + \frac{3p(p-1)H^2}{2}|x(t)|^{p-2}[1+|x(t)|^{2m_2} + |x(t-\omega(t))|^{2m_2}] \\ & \quad + k_6 + k_7|x(t)|^m + k_8|x(t-\omega(t))|^m\Big)\mathrm{d}t.\end{aligned} \tag{32}$$

Using the Young inequality, we have

$$|x(t)|^{p-1}|x(t-\omega(t))|^{m_1} \leq |x(t)|^{p+m_1-1} + |x(t-\omega(t))|^{p+m_1-1}. \tag{33}$$

Therefore, it can be inferred from (31)-(33) that

$$\big|\mathbb{E}|x(t_2)|^p - \mathbb{E}|x(t_1)|^p\big| \leq D_3(t_2-t_1), \tag{34}$$

where

$$D_3 = k_6 + (2pk_5 + k_7 + k_8 + 4pH + 6p(p-1)H^2)\left(\sup_{-\eta \le t < \infty} \mathbb{E}|x(t)|^m\right) < \infty. \tag{35}$$

Hence, $\mathbb{E}|x(t)|^p$ is uniformly continuous in t. Consequently, we conclude that $\lim_{t\to\infty} \mathbb{E}|x(t)|^p = 0$ according to (13). □

4 Conclusion

In this paper, we focused on the stabilization of HSDDEs with Lévy noise via delay feedback control, especially for those that do not conform to linear growth conditions. Considering the H_∞ and asymptotic stability of the controlled highly nonlinear HSDDEs driven by Lévy noise. By employing a Lyapunov functional, we provided several sufficient conditions that ensure H_∞-stability and asymptotic stability for the hybrid controlled system. We have shown that the designed controller stabilizes the unstable highly nonlinear HSDDEs system.

Acknowledgements

The author extends his appreciation to the anonymous referees for their very helpful suggestions which greatly improve the paper. This research work was supported by the Program of Young Scholar for Henan Polytechnic University (2020XQG-03).

References

[1] D. Applebaum, *Lévy Processes and Stochastic Calculus*, 2nd Edition, Cambridge University Press, 2009.

[2] D. Applebaum, M. Siakalli, *Stochastic stabilization of dynamical systems using Lévy noise*, Stoch. Dyn. **10** (2010) 509–527.

[3] A. Bahar, X. Mao, *Stochastic delay population dynamics*, Int. J. Pure Appl. Math. **11** (2004) 377–400.

[4] M. Fatehi Nia, E. Mirzavand, *Stochastic dynamics of Izhikevich-Fitzhugh neuron model*, J. Math. Model. **12** (2024) 199–214.

[5] C. Fei, W. Fei, X. Mao, M. Shen, L. Yan, *Stability analysis of highly nonlinear hybrid multiple-delay stochastic differential equations*, J. Appl. Anal. Comput. **9** (2019) 1053–1070.

[6] D. He, L. Xu, *Boundedness analysis of stochastic delay differential equations with Lévy noise*, Appl. Math. Comput. **421** (2022) 126902.

[7] L. Hu, X. Mao, Y. Shen, *Stability and boundedness of nonlinear hybrid stochastic differential delay equations*, Syst. Control Lett. **62** (2013) 178–187.

[8] L. Huang, X. Mao, *Delay-dependent exponential stability of neutral stochastic delay systems*, IEEE Trans. Autom. Control. **54** (2009) 147–152.

[9] M. Loeve, *Probability theory*, D. Van Nostrand Company, Inc, 1955.

[10] G. Li, *Stabilization of stochastic regime-switching Poisson jump equations by delay feedback control*, J. Inequal. Appl. **1** (2022) 20.

[11] M. Li, F. Deng, *Almost sure stability with general decay rate of neutral stochastic delayed hybrid systems with Lévy noise*, Nonlinear Anal. Hybrid Syst. **24** (2017) 171–185.

[12] G. Li, Z. Hu, F. Deng, H. Zhang, *Stabilization via delay feedback for highly nonlinear stochastic time-varying delay systems with Markovian switching and Poisson jump*, Electron. J. Qual. Theory Differ. Equ. **49** (2022) 1–20.

[13] W. Li, C. Fei, C. Mei, W. Fei, X. Mao, *Delay tolerance for stable hybrid stochastic differential equations with Lévy noise based on Razumikhin technique*, Syst. Control Lett. **176** (2023) 105530.

[14] X. Li, X. Mao, *Stabilisation of highly nonlinear hybrid stochastic differential delay equations by delay feedback control*, Automatica **112** (2020) 108657.

[15] G. Li, Q. Yang, *Stability analysis between the hybrid stochastic delay differential equations with jumps and the Euler-Maruyama method*, J. Appl. Anal. Comput. **11** (2021) 1259–1272.

[16] G. Li, Q. Yang, *Stabilization of hybrid stochastic systems with Lévy noise by discrete-time feedback control*, Int. J. Control. **95** (2020) 197–205.

[17] D. Liu, W. Wang, J. Menaldi, *Almost sure asymptotic stabilization of differential equations with time-varying delay by Lévy noise*, Nonlinear Dyn. **79** (2015) 163–172.

[18] M. Liu, Y. Zhu, *Stationary distribution and ergodicity of a stochastic hybrid competition model with Lévy jumps*, Nonlinear Anal. Hybrid Syst. **30** (2018) 225–239.

[19] Z. Lu, J. Hu, X. Mao, *Stabilisation by delay feedback control for highly nonlinear hybrid stochastic differential equations*, Discrete Contin. Dyn. Syst. Ser. B. **24** (2019) 4099–4116.

[20] X. Mao, *Stability of stochastic differential equations with Markovian switching*, Stoch. Proc. Appl. **79** (1999) 45–67.

[21] W. Mao, L. Hu, X. Mao, *The asymptotic stability of hybrid stochastic systems with pantograph delay and non-Gaussian Lévy noise*, J. Franklin Inst. **357** (2020) 1174–1198.

[22] X. Mao, A. Matasov, A. Piunovskiy, *Stochastic differential delay equations with Markovian switching*, Bernoulli. **6** (2000) 73–90.

[23] X. Mao, L. Shaikhet, *Delay-dependent stability criteria for stochastic differential delay equations with Markovian switching*, Stab. Control Theory Appl. **3** (2000) 88–102.

[24] X. Mao, C. Yuan, *Stochastic Differential Equations with Markovian Switching*, Imperial College Press, London, 2006.

[25] M. Rhaima, L. Mchiri, A. Makhlouf, *H_∞ and Asymptotic Stability via delay feedback for hybrid neutral stochastic delay differential equations with Lévy noise*, IMA J. Math. Control Inform. **40** (2023) 106–132.

[26] T. Tian, K. Burrage, P. Burrage, M. Carletti, *Stochastic delay differential equations for genetic regulatory networks*, J. Comput. Appl. Math. **205** (2007) 696–707.

[27] F. Wan, P. Hu, H. Chen, *Stability analysis of neutral stochastic dfferential delay equations driven by Lévy noises*, Appl. Math. Comput. **375** (2020) 125080.

[28] L. Wu, X. Su, P. Shi, *Sliding mode control with bounded L_2 gain performance of Markovian jump singular time-delay systems*, Automatica. **48** (2012) 1929–1933.

[29] A. Wu, H. Yu, Z. Zeng, *Variable-delay feedback control for stabilisation of highly nonlinear hybrid stochastic neural networks with time-varying delays*, Int. J. Control. **97** (2023) 744–755.

[30] C. Yuan, X. Mao, *Stability of stochastic delay hybrid systems with jumps*, Eur. J. Control. **6** (2010) 595–608.

[31] Q. Zhu, *Asymptotic stability in the pth moment for stochastic differential equations with Lévy noise*, J. Math. Anal. Appl. **416** (2014) 126-142.