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A p-Laplacian model for uneven illumination enhancement of document images

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Abstract. The exponential growth of low-cost digital imagery is latterly observed. Images acquired under uneven lighting are prone to experience poor visibility, which may severely limit the performance of most computational photography and automatic visual recognition applications. Different from current optimization techniques, we design a novel partial differential equation-based model to rectify the variable illumination artifacts. In this study, a large number of document samples capturing uneven illumination and low contrast conditions are tested to compare the effectiveness of the proposed local and nonlocal approaches.

Keywords: Document image processing, illumination correction, nonlinear diffusion, nonlocal p-Laplacian.
AMS Subject Classification 2010: 35A15, 94A08.

1 Introduction

With the current digital revolution and the availability of a wide range of image-capturing devices such as digital cameras, digital camcorders, smartphones, and scanners, document imaging has replaced paper files and documents as a vital medium to transmit information in people's life. However, by virtue of the effect of the shooting environment, the document images may undergo various distortions.

Bad illumination is among the major factors affecting image formation. It naturally leads to uneven global brightness due to the dissimilar texture of the object surface and the shadows produced from distinct light source directions. A non-uniform light distribution, unstable lighting, and the shade of other objects in the scene represent mainly the nonideal conditions under which unevenly illuminated images are acquired. This distortion leads to extreme quality degradation of document images and may

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confuse the interpretation of their content which necessarily poses significant and notable problems in the subsequent layout analysis and the character recognition stages. Consequently, correcting illumination on document images can considerably enhance the performance of subsequent analysis tasks.

Non-uniform illumination enhancement shines in many vision-based applications. Within the context of medical imaging, Grisan et al. [11] proposed an effective algorithm to model and correct the local luminosity and the variability of the contrast in non-uniformly illuminated Retinal images, whereas in [14], a shading correction procedure of the digital microscopy images is outlined, it is based upon the intrinsic properties of the image, which are revealed through Gaussian smoothing. Illumination correction is also a crucial step in pre-processing high-resolution remote sensing data for forest change detection studies [22], satellite images [6] or for remote sensing images covered by thin clouds [20]. Another of the most important problems of illumination normalization is face recognition under varying illumination [3] and object tracking in video sequences [16].

The goal of illumination enhancement algorithms is to weaken as much as possible the effects of the present shade or bright light. As part of document images processing, traditional procedures commonly adopt adaptive binarization methods [8, 19] to eliminate the shadow. Knowing that the lighting of a document image often varies slowly, homomorphic filters [10] can be employed to subtract the background from the initial image, these filters produce satisfactory outputs for the textual parts of the images, but unfortunately can damage photographic regions. The illumination-balance algorithm [13] can efficiently improve the quality of degraded document images with illustrations obtained by scanners. This technique translates into four distinct stages: edge detection step using Sbel edge detector in several directions, object mark step, light evaluation step, and illumination balance step. This technique is then improved in [1]. Meng et al [15] defined a Convex Hull to estimate the shading for scanned document images.

The crucial objective of this work is to present an improved nonlocal approach for estimating variable illumination in document images. From a physical standpoint, nonlocal approaches play a vital role in characterizing many natural phenomena. The concern for nonlocal methods is motivated by the ability of these approaches to capture with rigorous accuracy the effects that are difficult to describe by local models. Nonlocal functionals, nonlocal operators, and nonlocal problems defined in nonlocal function spaces, have gradually attracted the mathematical community's attention by its theoretical wealth, as for its concrete real-world applications. This type of model occurs in a quite natural way in many different contexts, such as, among others, continuum theory [21], physics-based nonlocal elasticity [5], machine learning [18] and phase transition [7], and so on.

To tackle the problem of varying illumination in document images, we introduce in this paper an effective nonlocal p-Laplacian based equation to estimate the illumination component of the degraded source image. The proposed approach inherits the advantages of the nonlocal models in preserving text textures and small details.

2 The proposed model

In this section, we formulate a nonlocal evolution equation to correct the document images acquired under variable illumination. Starting with a grayscale image, our evolution equation estimates the non-uniform illumination to offer a perfectly clean version of the initial image.

2.1 Derivation of the proposed model

According to Reintal-Cortex theory [12], a given image "U" can be decomposed as follows

$$U(x,y) \;=\; I(x,y)R(x,y).$$

$I(x,y)$ stands for the illuminated part of ambient light and $R(x,y)$ represents the reflected constituent of the real color of the object.

The aim of this work is to eliminate the effect of the component "I" and acquire the text component of the original document image. For illumination estimation, Ait Bella et al. [2] explain that when the image contains only text, the intensity of the variable illumination background takes on values greater than the intensity of the dark text, which allows approximating the illumination by replacing each pixel with the maximum average of its neighboring pixels. In essence, if I_{ij}^n represents the intensity of a pixel (i,j) at the n^{th} iteration, we get:

$$I_{ij}^{n+1} = \max\left\{I_{ij}^n, \frac{I_{i+1j}^n + I_{i-1j}^n}{2}, \frac{I_{ij+1}^n + I_{ij-1}^n}{2}, \frac{I_{i+1j+1}^n + I_{i-1j-1}^n}{2}, \frac{I_{i-1j+1}^n + I_{i+1j-1}^n}{2}\right\}. \tag{1}$$

By subtracting I_{ij}^n from both sides of Eq. (1), the discrete second derivatives in all directions appear and we obtain the discretized version of the equation

$$\frac{\partial I}{\partial t} \;=\; \frac{1}{2}\max\left\{0,\; \frac{\partial^2 I}{\partial x^2}, \frac{\partial^2 I}{\partial y^2}, \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} + 2\frac{\partial^2 I}{\partial x \partial y}, \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} - 2\frac{\partial^2 I}{\partial x \partial y}\right\}.$$

Similarly, taking into account only the vertical and horizontal directions, one can directly consider the Laplacian operator instead of the second derivatives.

Inspired by [2], we propose the following nonlinear equation as the diffusion process of our illumination estimation model:

$$\begin{cases} \frac{\partial I}{\partial t} = \max\{0, \operatorname{div}(|\nabla I|^{p-2}\nabla I)\}, & \text{in} \quad (0,T)\times\Omega, \\ \frac{\partial I}{\partial n} = 0, & \text{on} \quad \partial\Omega\times(0,T), \\ I(.,0) = I_0 & \text{in,} \quad\quad \Omega, \end{cases} \tag{2}$$

where $1 < p < \infty$ and Ω is a bounded open subset of $\mathbb{R}^2$. The p-Laplacian $\Delta^p I = \operatorname{div}(|\nabla I|^{p-2}\nabla I)$ has attracted much attention in various applications. For instance, it arises in non-Newtonian fluids, flows through porous media, reaction-diffusion problems, nonlinear elasticity, or in petroleum extraction.

It should be mentioned that when $p = 2$, the obtained results coincide with those in [2]. Our central idea is to adopt a non-linear generalization of the standard Laplacian for the sole purpose of correcting any non-uniform illumination effect by controlling the degree of smoothing by different choices of the exponent "p".

In this article, we deal with the evolutionary equation (2), we give an existence result for the solutions of the proposed model (2) within the framework of the theory of viscosity solutions [4].

2.1.1 Preliminaries

The theory of viscosity solutions applies to several partial differential equations, including the equation of the form

$$\frac{\partial I}{\partial t} + F(\nabla I, \nabla^2 I) = 0, \quad \text{in } Q_t = (0,T)\times\Omega, \tag{3}$$

where Ω is a domain in $\mathbb{R}^n$. F is a real-valued function on $\mathbb{R}^n \times S^n$, where S^n stands for the set of symmetric $n \times n$ matrices, ∇I and $\nabla^2 I$ denote respectively, the gradient and the Hessian matrix of I in space variables.

We appoint the hypotheses of F which are important to consider the viscosity solutions of (3):

- $(H1)$ F is continuous in $(\mathbb{R}^n \setminus \{0\}) \times S^n$.
- $(H2)$ F is degenarate elliptic, i.e, if $X \geq Y$, then $F(q,X) \leq F(q,Y)$ for all $q \in (\mathbb{R}^n \setminus \{0\})$.
- $(H3)$ $\mathscr{F}(F) \neq \emptyset$ where $\mathscr{F}(F)$ is the set of functions $f \in C^2[0,\infty)$ which satisfies.

$$f(0) = f^{'}(0) = f^{''}(0), \quad f^{''}(r) > 0, \quad \text{for all } r > 0,$$

and

$$\lim_{|x| \to 0, x \neq 0} F(\nabla f(|x|), \nabla^2 f(|x|)) = 0.$$

- $(H4)_+$ There exists $g \in \mathscr{G} := \{g \in C^2[0,\infty); g(0) = g'(0) = 0, g'(r) > 0 \; if \; r > 0, \lim_{r \to \infty} g(r) = \infty\}$ such that for all $A > 0$, there is $B > 0$, such that

$$F(\nabla(Ag(|x|)), \nabla^2(Ag(|x|))) \geq -B, \quad \text{for all } x \in \mathbb{R}^N \backslash \{0\}.$$

- $(H4)_-$ There exists $g \in \mathscr{G}$ such that for all $A > 0$, there is $B > 0$, such that

$$F(\nabla(-Ag(|x|)), \nabla^2(-Ag(|x|))) \leq B, \quad \text{for all } x \in \mathbb{R}^N \backslash \{0\}.$$

To determine viscosity solutions, it is necessary to introduce a class of “test functions”.

Definition 1. *[17] A function $\varphi \in C^2(Q_T)$ is admissible, in short $\varphi \in \mathscr{A}(F)$, if for any $\hat{z} = (\hat{t}, \hat{x}) \in Q_T$ with $\nabla \varphi(\hat{z}) = 0$, there exists a constant $\delta > 0$, $f \in \mathscr{F}(F)$ and $\omega \in C[0,\infty)$ satisfying $\omega \geq 0$ and $\lim_{r \to 0} \omega(r)/r = 0$, such that*

$$|\varphi(z) - \varphi(\hat{z}) - \frac{\partial \varphi}{\partial t}(\hat{z})(t - \hat{t}) \leq f(|x - \hat{x}|) + \omega(|t - \hat{t}|),$$

for all $z = (t,x)$ with $|z - \hat{z}| < \delta$.

Now we shall introduce a notion of viscosity solutions of (3).

Definition 2. *[17] Assume that $(H1)$ and $(H2)$ hold and that $\mathscr{F}(F)$ is not empty.*

1. *A function $u : Q_T \to \mathbb{R} \cup \{-\infty\}$ is a viscosity subsolution of (3) if $u^* < +\infty$ on $\bar{Q}_T$ and for all $\varphi \in \mathscr{A}(F)$ and all local maximum point z of $u^* - \varphi$ in Q_T ,*

$$\begin{cases} \frac{\partial \varphi}{\partial t}(z) + F(\nabla \varphi(z), \nabla^2 \varphi(z)) \leq 0, & if \quad \nabla \varphi(z) \neq 0, \\ \frac{\partial \varphi}{\partial t}(z) \leq 0, & otherwise. \end{cases}$$

2. *A function $u : Q_T \to \mathbb{R} \cup \{-\infty\}$ is a viscosity supersolution of* (3) *if $u_* > -\infty$ on $\bar{Q}_T$ and for all $\varphi \in \mathscr{A}(F)$ and all local minimum point z of $u_* - \varphi$ in Q_T ,*

$$\begin{cases} \frac{\partial \varphi}{\partial t}(z) + F(\nabla \varphi(z), \nabla^2 \varphi(z)) \ \geq \ 0, & \text{if} \quad \nabla \varphi(z) \neq 0, \\ \frac{\partial \varphi}{\partial t}(z) \ \geq \ 0, & \text{otherwise}. \end{cases}$$

3. *A function u is called a viscosity solution of* (3) *if u is both a viscosity sub- and super-solution of* (3).

2.1.2 Existence theorem

Theorem 1. *Let Ω be a regular bounded open subset of $\mathbb{R}^2$ and the initial data I_0 is bounded, uniformly continuous on $\mathbb{R}^2$. Then there exists a unique viscosity solution of the proposed model* (2).

Proof. In order to place Eq. (2) in the form (3), we rewrite

$$\begin{aligned} F(\nabla I, \nabla^2 I) &= -\max\left\{0, \operatorname{div}\left(|\nabla I|^{p-2}\nabla I\right)\right\} \\ &= -H\left(\operatorname{div}\left(|\nabla I|^{p-2}\nabla I\right)\right)\operatorname{div}\left(|\nabla I|^{p-2}\nabla I\right) \\ &= -H\left(|\nabla I|^{p-2}\operatorname{trace}\left(\left(Id + (p-2)\frac{\nabla I \otimes \nabla I}{|\nabla I|^2}\right)\nabla^2 I\right)\right) \\ &\quad \times |\nabla I|^{p-2}\operatorname{trace}\left(\left(Id + (p-2)\frac{\nabla I \otimes \nabla I}{|\nabla I|^2}\right)\nabla^2 I\right), \end{aligned}$$

where $\otimes$ denotes the tensor product, $Id = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, where H is the Heaviside step function, defined by

$$\forall\, x \in \mathbb{R}, \;\; H(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1, & \text{if } x \geq 0. \end{cases}$$

The function F satisfies the assumptions (H) listed above. In fact:

- F is continuous as composed of the two continuous functions:

$$\begin{cases} F_1 & : \mathbb{R}^2 \times S^2 \to \mathbb{R} \\ & (q, X) \mapsto -|q|^{p-2}\operatorname{trace}\left(\left(Id + (p-2)\frac{q \otimes q}{|q|^2}\right)X\right), \end{cases}$$

and

$$\begin{cases} F_2 & : \mathbb{R} \to]-\infty, 0] \\ & x \mapsto H(-x)x \end{cases}$$

- The degenerate ellipticity of F comes from the degenerate ellipticity of F_1 and the increasing nature of F_2.

- For $1 < p < 2$, we can easily verify that $f(r) = r^{1+\alpha}$ with $\alpha > 1/(p-1)$ is a function in $\mathscr{F}(F)$. Whereas, we select $f(r) = r^4 \in \mathscr{F}(F)$ for $p \geq 2$.

- Following [17], we also state that $g_1(r) = \frac{p-1}{p} r^{\frac{p-1}{p}} \in \mathscr{G}$ for $1 < p < 2$ and the function $g_2(r) = r - \arctan(r)$, for $p > 2$ satisfy $(H4)_{\pm}$.

According to Theorem 4.9 in [17], we obtain the existence of a unique viscosity solution $I \in UC([0,T) \times \mathbb{R}^2)$ to the following problem

$$\begin{cases} \frac{\partial I}{\partial t} + F(\nabla I, \nabla^2 I) = 0, & \text{in } (0,T) \times \mathbb{R}^2, \\ I(.,0) = I_0, & \text{in } \mathbb{R}^2. \end{cases}$$

In particular, this solution can be defined on $D \times [0,T)$, where $\Omega \subset\subset D$. By Perron's method, we obtain existence of a viscosity solution $\tilde{I}$ on $D \times [0,T)$, then with the localization property [9] $\tilde{I}\,|_{\Omega\times(0,T)}$ is a viscosity solution of the differential equation that satisfies the initial condition in the classical sense as well. The viscosity solution $\tilde{I}$ satisfies the differential equation on $\partial\Omega \times (0,T)$ and thus $\tilde{I}\,|_{\Omega\times(0,T)}$ satisfies the Neumann boundary condition in the viscosity sense. □

2.2 A nonlocal extension

The previous subsection provided the initial formulation of the proposed model. The goal is to restore and reconstruct the badly illuminated documents in such a way that the text will be easily readable on the output document images but since document images can contain even more redundancy than other forms of images, we propose to modify the equation (2) in an attempt to better restore degraded documents. To achieve this goal, a nonlocal strategy looks like a potential breakthrough.

In fact, the basic idea is to avail the self-similarity commonly present in natural as well as document images. Thus, to take benefit of the redundancy and self-similarity of the information in the document images, we propose the following nonlocal analogous problem to (2) with homogeneous Neumann boundary conditions:

$$\begin{cases} \frac{\partial I}{\partial t} = \max\{0, \Delta_{NL}^p I\}, & \text{in} \quad \Omega,\ t > 0, \\ I(x,0) = I_0, & \text{in} \quad \Omega, \end{cases} \tag{4}$$

where

$$\Delta_{NL}^p I := \int_{\Omega} J(x-y)|I(y,t) - I(x,t)|^{p-2}(I(y,t) - I(x,t))dy \tag{5}$$

is the nonlocal p-Laplacian operator, $J : \mathbb{R}^2 \to \mathbb{R}$ is a nonnegative continuous radial function and $1 < p < +\infty$.

The use of the nonlocal p-Laplacian (5) allows a powerful estimation process and since it does not rely on the gradient to extract the direction of diffusion, the proposed model proves capable of preserving the textures and details of the text.

3 Numerical results

This section is devoted to the experimental part to evaluate the effectiveness of our method. In order to numerically approach the problem (4), we choose the weight function J which appears in the definition of the nonlocal p-Laplacian as:

$$J(x) := \begin{cases} \exp\left(-\frac{1}{h^2}|x|^2\right), & if \;\; x < d, \\ 0, & if \;\; x \geq d, \end{cases}$$

where h and d are fixed positive constants.

We consider the following discretization of the nonlocal p-Laplacian operator

$$\triangle_{NL}^{p}(u_i) = \sum_{j \in \mathcal{N}_i} J_i |I_j - I_i|^{p-2}(I_j - I_i), \quad j \in N_i,$$

where I_i and J_i are respectively the value of a pixel i $(1 \leq i \leq N)$ and the discrete version of the weight function $J(i-j)$, $\mathcal{N}_i = \{j : |i-j| \leq d\}$ denotes the neighbors set of the pixel i.

The detailed steps of the proposed method are as follows:

- **Input :** The acquired image I_0,
- **Initialization :** We set $I^0 = \log(I_0 + 1)$, and choose $dt > 0$, $h > 0$, $k_1, k_2 > 0$,
- **Compute :** $I_i^{n+1} = I_i^n + dt \times \max\{0, \triangle_{NL}^{p(|\nabla I_0|)}(I_i^n)\} \quad i = 1, ..., N$,
- **Output :** The reflectance $R = \exp(I^0 - I^{n+1})$.

As the selection of the exponent "p" directly affects the efficiency of the proposed algorithm, this choice always raises questions. In this work, we propose to choose it depending on the local gradient of the initial image, such as:

$$p(|\nabla I_0|) = 1 + k_1 \exp(-k_2 |\nabla I_0|), \quad k_1, k_2 > 0.$$

This function is carefully chosen so that its value is strictly between 1 and 2 if illumination changes and shadows are severe and that the value of "p" is greater than or equal to 2 if illumination variations are regular.

The parameters for our nonlocal model are set as follows: $h = 80$, $k_1 = 5$ and $k_2 = 0.1$.

We have experimented on several document images with challenging illumination conditions. Then, we also experimented with the document images from Handwritten Document Image Binarization Contest (H-DIBCO) dataset.

Figs. 1-3 compare the results obtained using model 2 with $p = 2$, ie by considering the usual Laplacian operator with those of the proposed local p-Laplacian model and finally with the text recovered by the proposed nonlocal model 4. The consideration of the local Laplacian operator in the proposed model limits the performance of the proposed approach, we see that the effects of non-uniform illumination are not definitively eliminated, the proposed local model can get rid of these undesirable effects, while the proposed nonlocal model manages to eliminate all degradations while keeping all the details of the text. Fig. 4 asserts that the proposed nonlocal approach is also capable of enhancing bar codes and digits even if this type of image is generally very difficult to model.

The last figure shows the results obtained by the proposed nonlocal model using three selected document images with degrading illumination from the (H-DIBCO) dataset.

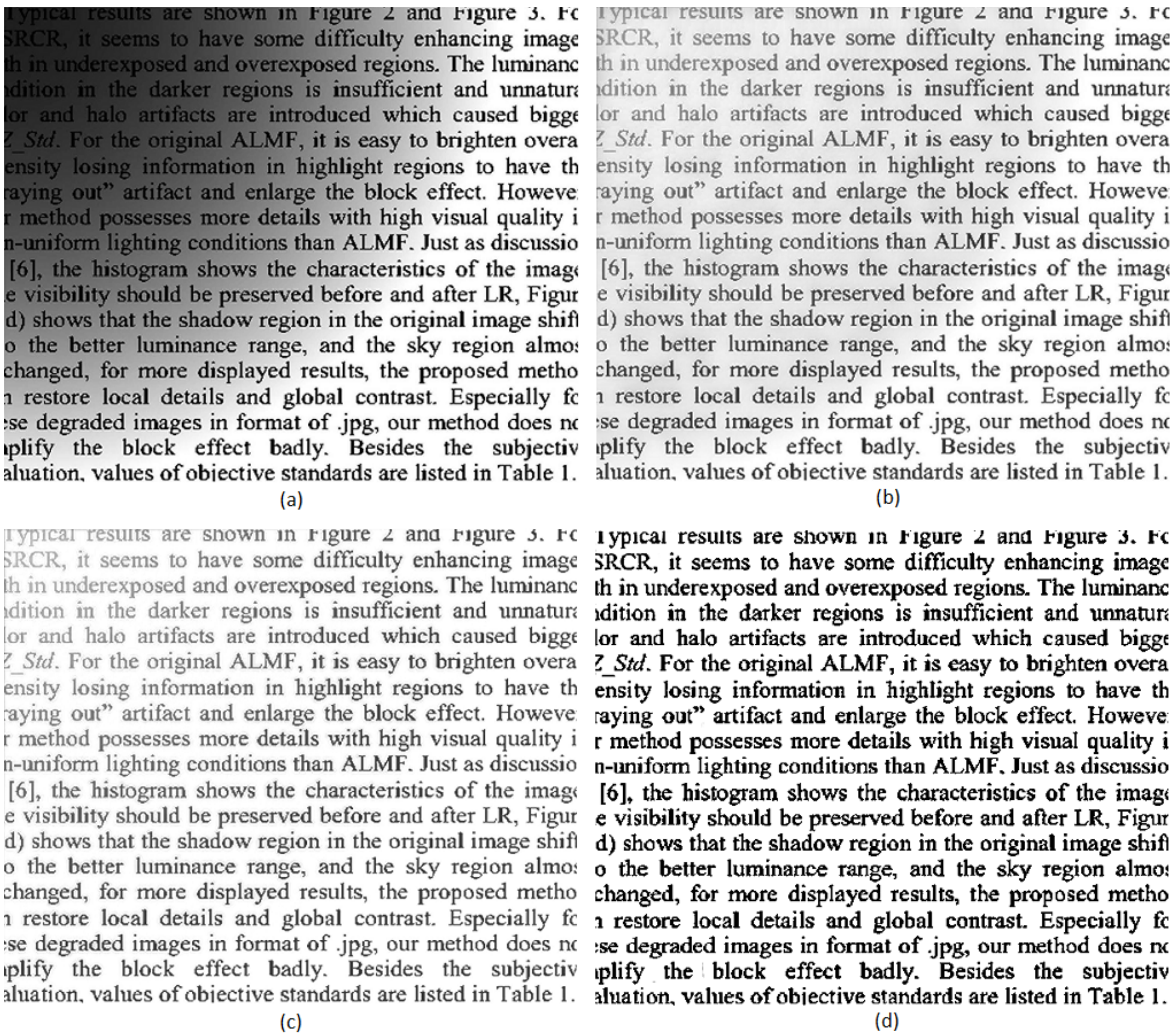

Figure 1: Images enhancement results: (a) Original degraded image, (b) Recovered reflectance by the proposed nonlocal model (2) with $p = 2$, (c) Recovered reflectance by the proposed nonlocal model (2), (d) Recovered reflectance by the proposed nonlocal model (4).

4 Conclusions

In this paper, we have introduced two novel document image enhancement approaches that can be applied to both handwritten documents and machine-printed images. The idea is essentially based on the estimation of the variation of background illumination of the degraded source image then we subtract the estimate component from the input document image to attain a comfortably readable text. We have proved an existence result for the solutions of the local proposed model within the framework of the theory of viscosity solutions and we have proposed a particular choice of the exponent "p" which adapts

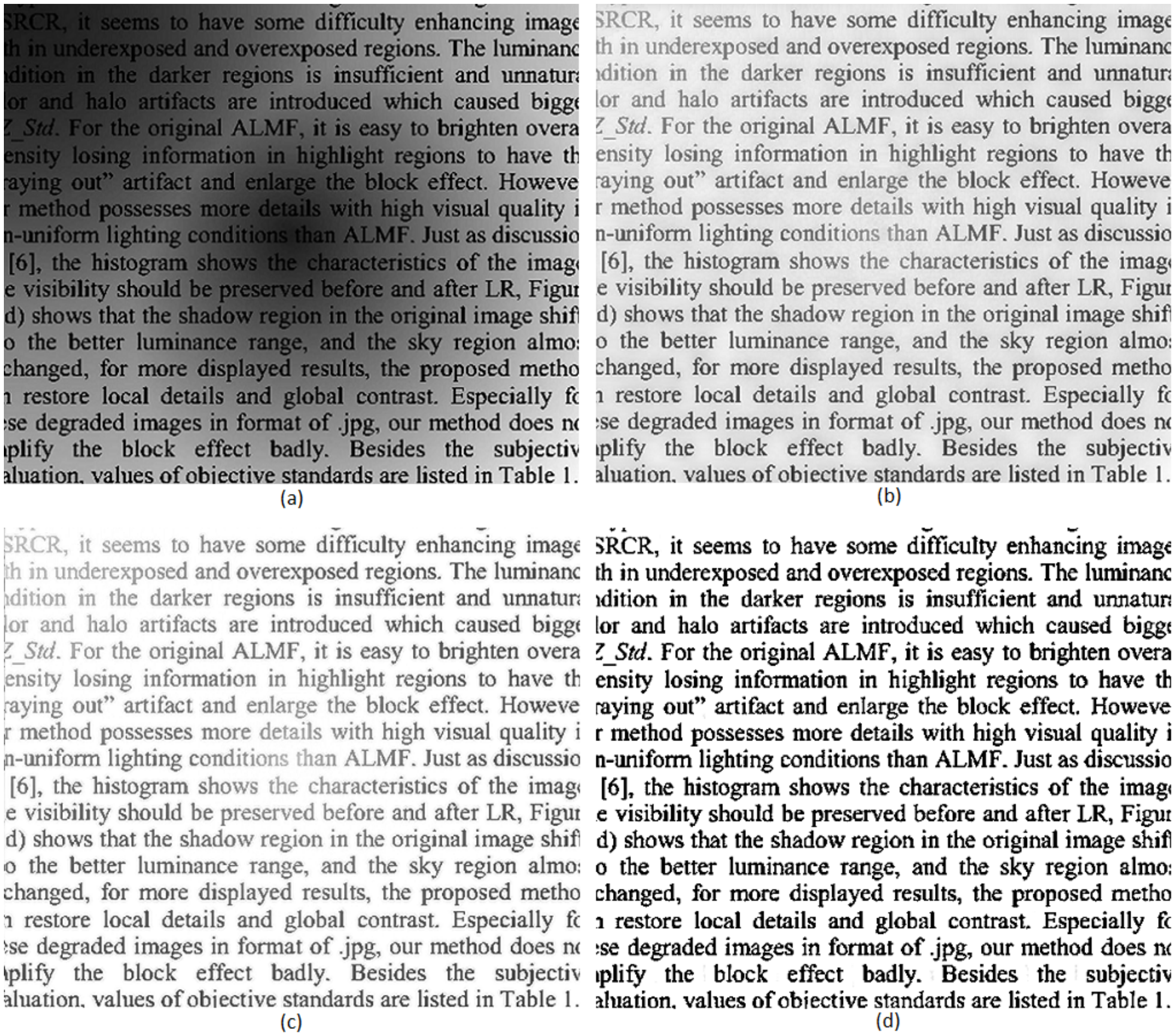

Figure 2: Images enhancement results: (a) Original degraded image, (b) Recovered reflectance by the proposed nonlocal model (2) with $p = 2$, (c) Recovered reflectance by the proposed nonlocal model (2), (d) Recovered reflectance by the proposed nonlocal model (4).

to any type of shade. Our proposed nonlocal approach shows outstanding performance in experiments.

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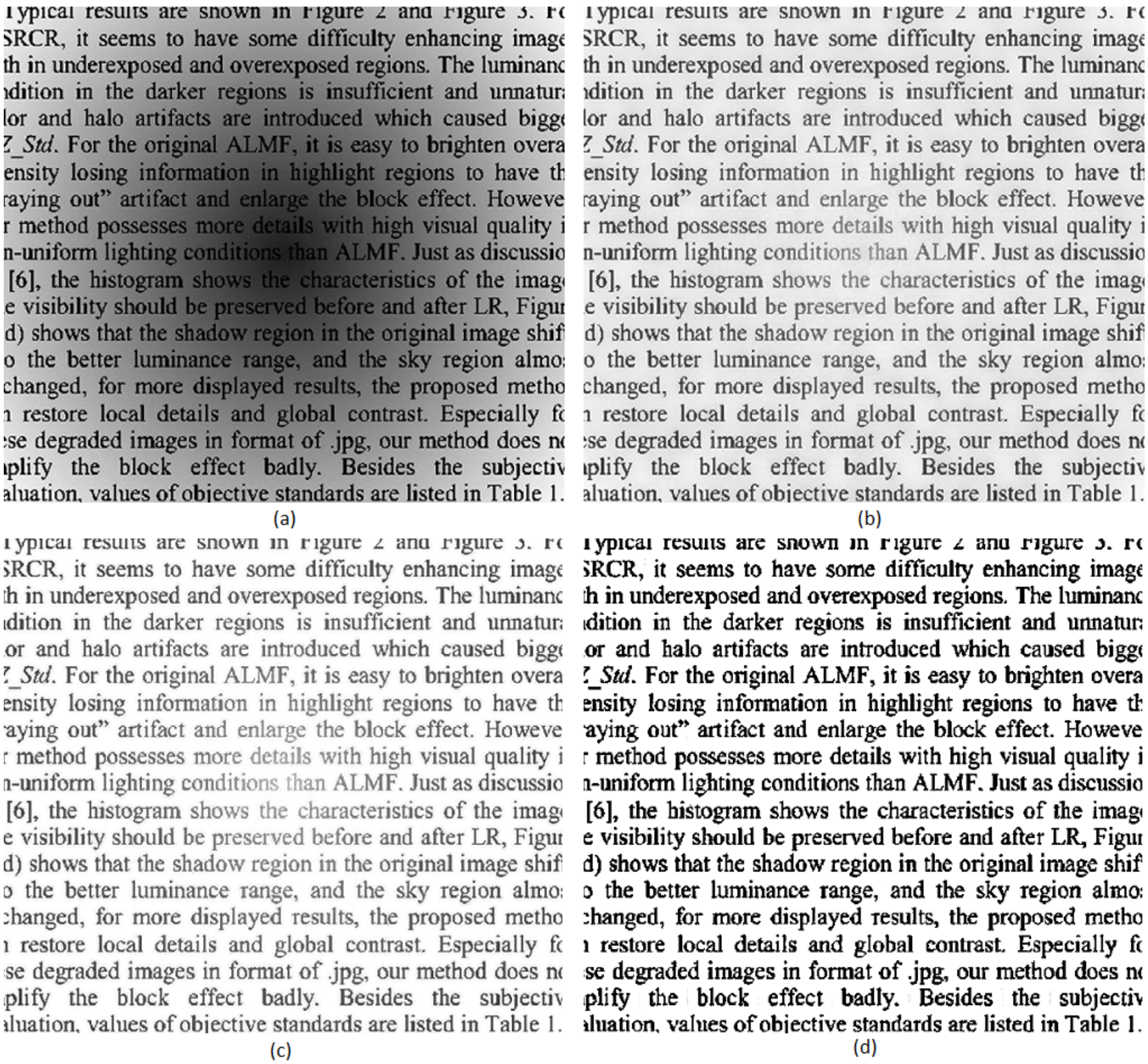

Figure 3: Images enhancement results: (a) Original degraded image, (b) Recovered reflectance by the proposed nonlocal model (2) with p=2, (c) Recovered reflectance by the proposed nonlocal model (2), (d) Recovered reflectance by the proposed nonlocal model (4).

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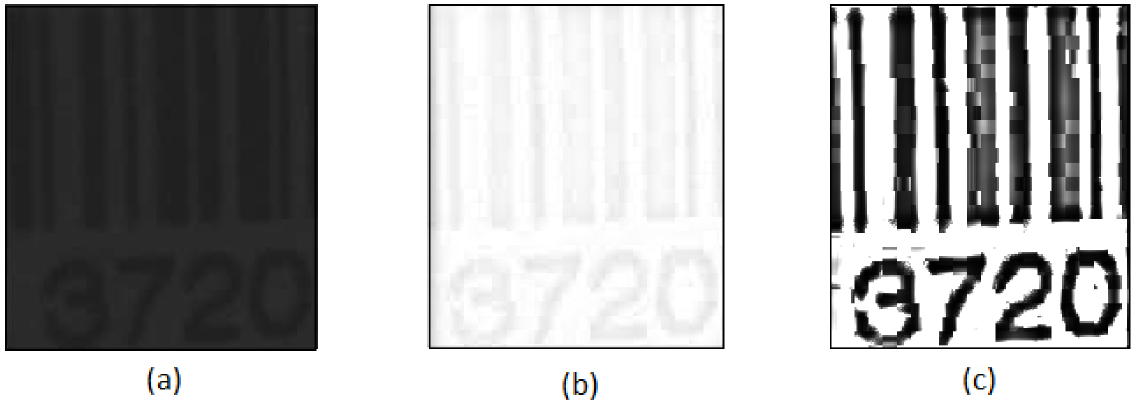

Figure 4: Images enhancement results: (a) Original degraded images, (b) Recovered digits by the proposed nonlocal model (2), (c) Recovered digits by the proposed nonlocal model (4),

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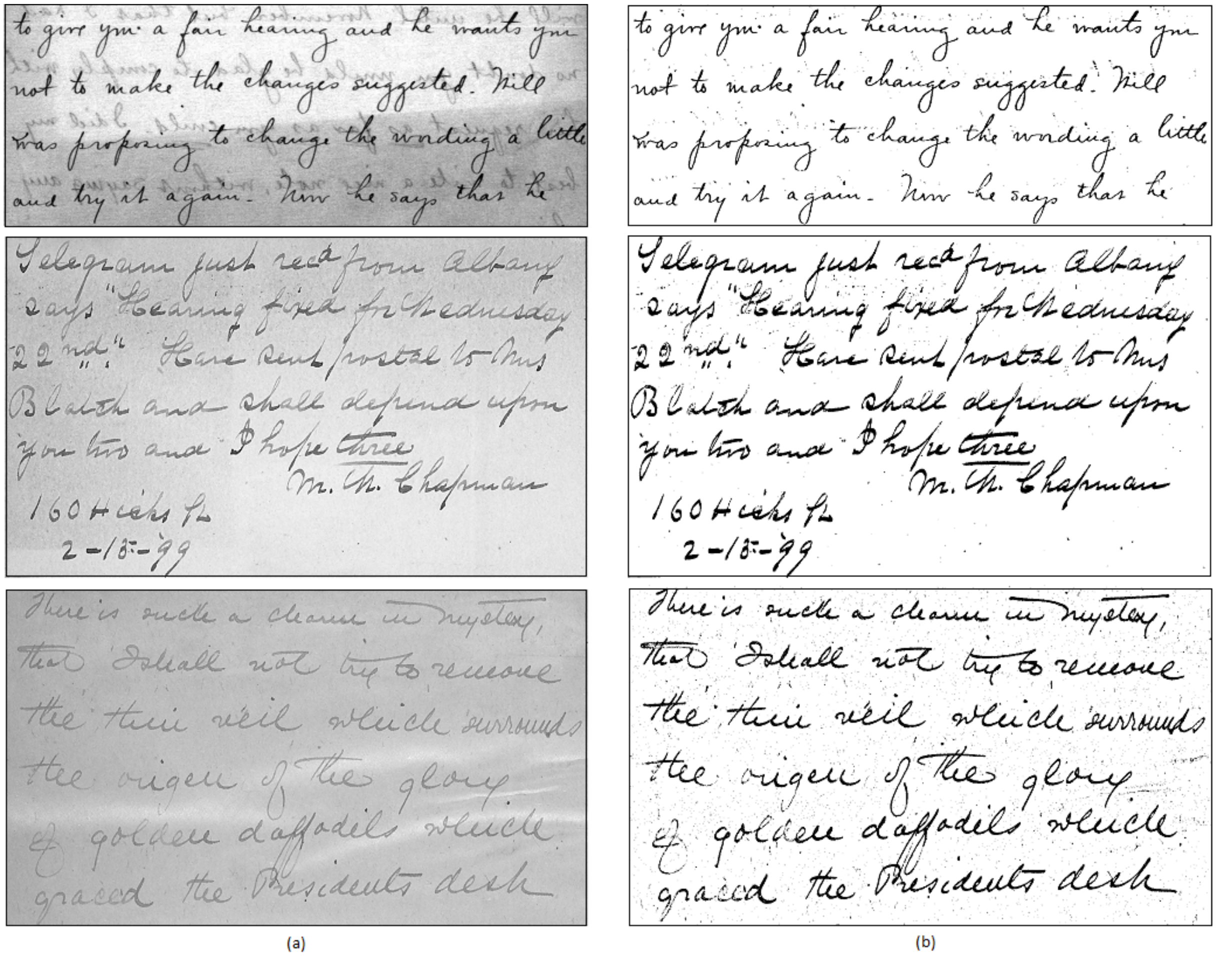

Figure 5: Images enhancement results from H-DIBCO: (a) Original degraded images, (b) Recovered text by the proposed nonlocal model (4).

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