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A bargaining game model for performance evaluation in network DEA considering shared inputs in the presence of undesirable outputs

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Abstract. Data Envelopment Analysis (DEA) is a non-parametric method for measuring the relative efficiency of peer decision-making units (DMUs), where the internal structures of DMUs are treated as a black box. Traditional DEA models do not pay attention to the internal structures and intermediate values. Network data envelopment analysis models addressed this shortcoming by considering intermediate measure. The results of two-stage DEA model not only provides an overall efficiency score for the entire process, but also yields an efficiency score for each of the individual stages. The centralized model has been widely used to evaluate the efficiency of two-stage systems, but the allocation problem of shared inputs and undesirable outputs has not been considered. The aim of this paper is to develop a method based on bargaining for evaluation in network DEA considering shared inputs and undesirable outputs. The two stages are considered as players to bargain for a better payoff, which is offered by DEA ratio efficiency score of DMUs. The efficiency model is developed as a cooperative game model. Finally, a numerical example is given to evaluate the proposed model.

Keywords: Data envelopment analysis, Nash bargaining theory, shared inputs, two-stage network, undesirable outputs.
AMS Subject Classification 2010: 34A34, 65L05.

1 Introduction

Data Envelopment Analysis (DEA) is a popular and much handled non-parametric frontier analysis tool for evaluating the relative performance of a set of homogeneous decision making units (DMUs) introduced by Charnes et al. [4]. In recent years, there have been extensive developments of DEA models. Tracy and Chen [36] introduced a generalized model for weight restrictions in DEA. Du et al. [10] introduced slacks-based measure (SBM) model to measure super-efficiency in DEA. Tone [35] proposed

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SBM model to measure efficiency in DEA, in which all inefficiencies are calculated. In traditional DEA models, the units are considered as black boxes and only their inputs and outputs are considered in the evaluation and their internal structure is not considered. Therefore, for inefficient units, the source of inefficiency is not specified. In fact, noticing the interior structure of the system, network DEA models can be a more suitable than the traditional DEA models.

In two-stage DEA modeling, the outputs of the first stage enter the system as the input of the second stage, which are called intermediate actions. Fare et al. [11] introduced network DEA models that examine the operation of processes and components in evaluating system performance. In recent years, many scholars (see, e.g., Seiford et al. [31]; Zhu [42]; Golany et al. [15]; Liang et al. [24]; Chen et al. [5]; Kao and Hwang [18] and [19], Wu [39]; Fukuyama and Weber [13]; Cook and Zhu [8] and Lozano [26]) applied the original DEA model for DMUs with two-stage network structure which does not address contrasts between the two stages happening for the intermediate measures. In other word, the second stage may have to reduce its inputs (intermediate measures) in order to gain an efficient case. In fact, noticing the interior structure of the system, the network DEA model can be a more suitable model than the traditional DEA model, which can particular more sensitively and representative information.

Cooperative and non-cooperative game theories are discussed in order to determine the upper and lower bounds of the efficiencies of sub-DMUs in different stages to assess the relative performance of the operational units. Wang et al. [37] applied a non-radial two-stage DEA to evaluate the innovation efficiency of new energy enterprises, and Li et al. [23] expanded an original two-stage network DEA to test the fire protection efficiency in the United States at a state level. However, since the above two-stage process involves a pure sequential system from the output of the first stage to the input of the second stage, and does not consider the intermediate undesirable output problem, they are not convenient for the real condition of this study. The current paper applies directly the Nash bargaining game theory to the efficiency of DMUs that have the aforementioned two-stage processes. We view both stages as two individuals bargaining with each other for a better payoff, which is characterized by the DEA ratio efficiency of each individual stage. In general, the resulting Nash bargaining game model is highly nonlinear, given the nature of ratio forms of DEA efficiency. The current paper shows that this nonlinear Nash bargaining model can be converted into a parametric linear programming problem with one parameter, whose lower and upper bounds can be determined. As a result, a global optimal solution can be found using a heuristic search on the single parameter. In our Nash bargaining game model, the breakdown or status quo point is determined via the standard DEA model. The bargaining efficiency scores of the both stages may depend on the selection of the breakdown point.

The major contributions of this paper are the model details, the internal structure and the interactions of two stages, thereby providing decision makers with specific, comprehensive information and allowing them to recognize the deficiencies in the overall system. Also, the offered model deals with the undesirable outputs and adds the feedback variable as the recyclable input.

The rest of this paper is arranged as follows. In Section 2, we provide some basic theoretical preliminaries and review the related literature. Section 3 develops the model and discusses the model solution method. Section 4 provides a numerical example and the conclusions are given in Section 5.

2 Theoretical preliminaries and literature review

In this section, we briefly review the main theoretical preliminaries as well as the related literature.

2.1 Theoretical preliminaries

2.1.1 Conventional DEA model

Assume that there are n DMUs, $(DMU_j,\ j = 1,2,\ldots,n)$ which consume m inputs $(x_i,\ i = 1,2,\ldots,m)$ to produce R outputs $(y_r : r = 1,2,\ldots,R)$. The best relative efficiency in each DMU is determined by the following model (DMUo relative efficiency):

$$
\begin{aligned}
\theta_o^* = \max\ & \sum_{r=1}^{R} u_r y_{ro} \\
\text{subject to :}\ & \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{R} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, && j = 1,2,\ldots,n, \\
& v_i \ge 0, && i = 1,\ldots,m, \\
& u_r \ge 0, && r = 1,\ldots,R.
\end{aligned}
\tag{1}
$$

In this model, v_i is the weights of the inputs and u_r is the weights of the outputs. DMUo is efficient when the relative efficiency in the above model equals to 1. One of the drawbacks of model (1) is that it is not designed to measure the efficiency of internal processes in a DMU. Two-stage DEA models have been proposed to handle this issue.

2.1.2 Structure of two-stage DEA models

Suppose we have n DMUs with two-stage structure. Each DMU is denoted by DMU_j $(j = 1,\ldots,n)$. Stage 1 of DMU_j consumes m inputs X_{ij} $(i = 1,\ldots,m)$ to produce intermediate measures Z_{dj} $(d = 1,\ldots,D)$. Then outputs Y_{rj} $(r = 1,\ldots,R)$ generated by consuming intermediate measures Z_{dj} $(d = 1,\ldots,D)$ to stage 2. The structure of a two-stage system is depicted in Fig. 1.

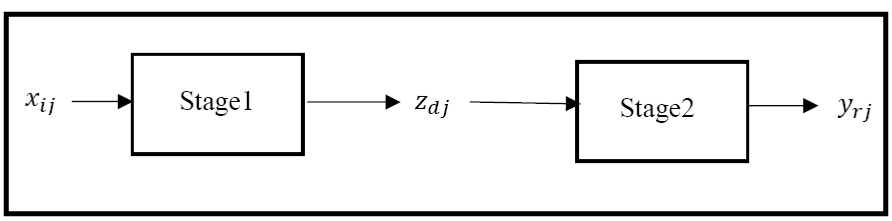

Figure 1: Two-stage production system.

Assume that v, u, w are the weight vector of inputs, outputs and intermediate products, respectively. Kao and Hwang [19] presented the following model for evaluating the efficiency of DMUo (DMU under evaluation) to measure the overall efficiency of the system and the efficiency of stages under constant return to scale (CRS) assumption, simultaneously:

$$e_O^S = \max\ \sum_{r=1}^{R} u_r y_{ro}$$

$$\begin{aligned} \text{subject to}: & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{R} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \\ & \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \\ & v_i, u_r, w_d \geq 0, \quad i = 1,\dots,m, \quad r = 1,\dots,R, \quad d = 1,\dots,D. \end{aligned} \tag{2}$$

Let $(u*, v*, w*)$ be an optimal solution of this model. In this case, we have

$$e_o^s = e_o^1 \cdot e_o^2 = \ \max \frac{\sum_{r=1}^{R} u*_r y_{ro}}{\sum_{i=1}^{m} v*_i x_{io}}, \qquad e_o^1 = \max \frac{\sum_{d=1}^{D} w*_d z_{do}}{\sum_{i=1}^{m} v*_i x_{io}}, \quad e_o^2 = \max \frac{\sum_{r=1}^{R} u*_r y_{ro}}{\sum_{d=1}^{D} w*_d z_{do}}.$$

Here e_o^s , e_o^1 and e_o^2 indicate the overall efficiency of the system and efficiency of the first and second stages, respectively.

2.1.3 Nash bargaining game method

Denote the set of all individuals by $N = \{1, 2, \dots, n\}$ and a payoff vector is an element of the payoff space R^N, which is the n-dimensional Euclidean space indexed by the set of individuals. A feasible set S is a subset of the payoff space, and a breakdown point $b \in R^2$ is an element of the payoff space. A bargaining problem can be then specified as the triple $\left(N,\ S,\ \overrightarrow{b}\right)$ consisting of individuals, feasible set and breakdown point. Nash [30] required that the feasible set is compact, convex and contains some payoff vector such that each individual's payoff is greater than the individual's breakdown payoff. The solution is a function F that is associated with each bargaining problem $\left(N,\ S,\ \overrightarrow{b}\right)$, expressed as $F\left(N,\ S,\ \overrightarrow{b}\right)$. Nash [30] argued that a reasonable solution should satisfy the following four attributes:

1. *Pareto efficiency*,
2. *Invariance with respect to affine transformation*,
3. *Independence of irrelevant alternatives*,
4. *Symmetry.*

These four properties are well known and discussed extensively in the literature. For the traditional bargaining problem, Nash [30] has shown that there exists a unique solution that satisfies the above four properties, called the Nash solution and the solution can be obtained by solving the following maximization problem

$$\max_{\text{vec}u \in \text{vec}S, \text{vec}u \geq \text{vec}b} \prod_{i=1}^{n} (u_i - b_i), \tag{3}$$

where $\overrightarrow{u}$ is the payment vector of individuals and u_i is the ith element of $\overrightarrow{u}$, b_i is an element of $\overrightarrow{b}$. To begin bargaining, we need to detect a breakdown point for the two players. The breakdown point shows the possible payoff pairs obtained if one determines not to bargain with the other player.

2.2 Literature review

In this section, we briefly review the literature two-stage DEA and Nash bargaining modeling.

2.2.1 Two-stage DEA and undesirable outputs

Standard DEA models consider DMUs being analyzed as black boxes where inputs are consumed to produce outputs (Avkiran [2]). As a result, single-stage DEA models could deliver inaccurate evaluations of efficiency. On the other hand, two-stage DEA models were designed to account for the internal processes and structure of the DMUs, allowing the decision makers to identify the sources of inefficiency across the sub-DMUs composing a given DMU (Kao and Hwang [18]; Fare and Grosskopf [11]). Khalili-Damghani et al. [17] offered a new network DEA model for measuring the performance of agility in supply chains. Moreover, the two-stage DEA has been found to be an effective approach for expressing such internal relationship. Kao and Liu [22] applied the relational network DEA approach to a two-stage system with fuzzy data, while Kao and Lin [20] considered a system with parallel processes and fuzzy data. With all that being said, there are some DEA studies of undesirable outputs in two-stage frameworks. In the context of network DEA with undesirable output, Fukuyama and Weber [13] considered undesirable outputs for evaluation of bank efficiency in a two-stage series system. Hu et al. [16] discussed the ecological utilization of leather tannery waste in the leather industry. Lozano and Gutirrez [27] proposed a distance approach to deal with network DEA problems in which undesirable outputs are generated. Shakouri et al. [32] proposed a stochastic p-robust approach to two-stage network DEA model. They obtained an ideal robustness level and the maximum possible overall efficiency score of each DMU over all permissible uncertainties and also the minimal amount of uncertainty level for each DMU under their proposed models. Maghbouli et al. [28] proposed a network DEA model with undesirable intermediate products. In their study, the undesirable intermediate products are studied either as final outputs or as intermediate outputs used as inputs to the next stage. Song et al. [33] carried out a systematic study of SBM model considering undesirable outputs and further expanded SBM model from the perspective of two-stage networks. Wang et al. [38] studied the efficiency of the Chinese commercial banking system in a two-stage network with undesirable non-performing loans. Also, Wu et al. [41] presented a method for analyzing the reuse of undesirable intermediate outputs in a two-stage production process with a shared resource. Shared resources are input resources that not only were used by both the first and second stages, but also had the property that the proportion used in each stage could not be conveniently splitted up and allocated to the operations of the two stages. Fathalikhani [12] proposed a two-stage DEA model considering simultaneously the structure of shared inputs, additional input in the second stage and part of intermediate products as the final output. In addition, a part of the second stage outputs is undesirable which can be feedback as raw materials to the first stage.

2.2.2 Barganing game model

The centralized model is specified by the concept of cooperative game theory, where the two stages can be considered as players to maximize the efficiency of the whole system. Tavana et al. [34] , Chu et al. [7] and Mahmoudi et al. [29] proposed DEA models in the bargaining game structure for efficiency measurement of DMUs. Liang et al. [24] suggested obtaining the maximum achievable efficiency of one stage firstly, and computing the efficiency of the other stage subsequently. In recent literature, some researchers suggested methods for measuring efficiency of DMUs in the bargaining game with two-stage network structures (see, e.g. Wu et al. [40], Du et al. [9] and Zhou et al. [43]). Chen et al. [6], Lim and Zhu [25], Galagedera [14] and Kao and Liu [21] proposed different models for measuring efficiency of network DEA. Finally, in a recent study, Abdali and Fallahnejad [1] developed a bargaining game model for measuring the efficiency of DMUs that have a two-stage network structure with non-discretionary inputs.

3 Nash bargaining game model

3.1 The proposed two-stage DEA model

The below Fig. 2 illustrates a two-stage network process situation, where each DMU is composed of two sub-DMUs sequentially. Considering homogeneous DMUs denoted by $DMU_j\,(j=1,\ldots,n)$, and that each DMU_j has a two-stage internal structure as shown in Fig. 2, the first stage and the second stage are connected in series. We denote, for each $DMU_j\,(j=1,\ldots,n)$, $x_{ij}^{(1)}\,(i=1,\ldots,I_1)$ and $\alpha x_{ij}^{(2)}\,(i=1,\ldots,I_2)$ as the two inputs to stage 1. The outputs of stage 1 are denoted by $y_{rj}^{bad}\,(r=1,\ldots,R)$ and $z_{dj}\,(d=1,\ldots,D)$ and the outputs of stage 2 are denoted by $y_{rj}^{good}(r=1,\ldots,R)$. We denote the intermediate measures by $z_{dj}\,(d=1,\ldots,D)$, that are D outputs of the first stage and inputs to the second stage. In the stage 2, we also denote $\overline{\alpha} x_{ij}^{(2)}\,(i=1,\ldots,I_2)$ and $y_{rj}^{good}\,(r=1,\ldots,R)$ as inputs and outputs. Here, there is undesirable (bad) output that can significantly decrease or increase the efficiency of two-stage network DEA. Since the operator of DMU_j can freely assign the shared input resources between the two stages, the shared input resources are considered as an extra input, so that DMU_j allocate some parts α of it to the first stage and the remaining $(1-\alpha)=\overline{\alpha}$ to the second stage.

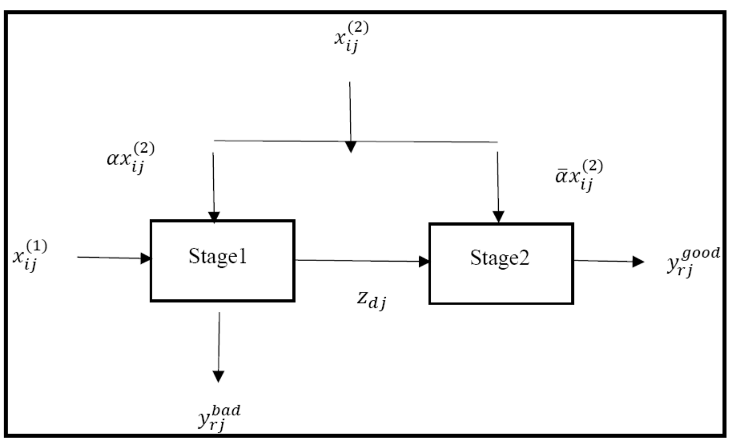

Figure 2: The two-stage network DEA structure with shared inputs and undesirable outputs.

Based upon the CRS model introduced by Charnes et al.1, for each $DMU_j\,(j=1,\ldots,n)$, we introduce e_j^1 and e_j^2 as the efficiency scores of stage 1 and stage 2, respectively:

$$e_j^1=\frac{\sum_{d=1}^{D} w_d\, z_{dj}+\sum_{r=1}^{R} u_r^{bad}\, y_{rj}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)}\, x_{ij}^{(1)}+\alpha\sum_{i=1}^{I_2} v_i^{(2)}\, x_{ij}^{(2)}}\leq 1, \tag{4}$$

and

$$e_j^2 = \frac{\sum_{r=1}^{R} u_r^{good} y_{rj}^{good}}{\sum_{d=1}^{D} w_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{ij}^{(2)}} \leq 1. \tag{5}$$

Then, $E = e_j^1 . e_j^2$ is the overall efficiency of the whole two-stage network DEA. The two-stage system is efficient if and only if $e_j^1 = e_j^2 = 1$. Here, considering the two-stage network as a cooperative game model, we define stage 1 and stage 2 as follows:

$$\begin{aligned} e_{min}^1 = \max \ & \frac{\sum_{d=1}^{D} w_d z_{do} + \sum_{r=1}^{R} u_r^{bad} y_{ro}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} \\ \text{subject to}: \ & \frac{\sum_{d=1}^{D} w_d z_{dj} + \sum_{r=1}^{R} u_r^{bad} y_{rj}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{ij}^{(2)}} \leq 1, \\ & w_d, u_r^{bad}, v_i^{(1)}, v_i^{(2)} \geq \varepsilon, \quad i = 1, \ldots, m; \quad r = 1, \ldots, R; \quad d = 1, \ldots, D, \end{aligned} \tag{6}$$

and

$$\begin{aligned} e_{min}^2 = \max \ & \frac{\sum_{r=1}^{R} u_r^{good} y_{ro}^{good}}{\sum_{d=1}^{D} w_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} \\ \text{subject to}: \ & \frac{\sum_{r=1}^{R} u_r^{good} y_{rj}^{good}}{\sum_{d=1}^{D} w_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{ij}^{(2)}} \leq 1, \\ & w_d, u_r^{good}, v_i^{(2)} \geq \varepsilon, \quad i = 1, \ldots, m; \quad r = 1, \ldots, R; \quad d = 1, \ldots, D. \end{aligned} \tag{7}$$

In the above models, the two stages can be considered as two players in the cooperative game model. The $v_i^{(1)}$ is the weight attributed to input $x_{io}^{(1)}$, $v_i^{(2)}$ is the weight attributed to input $x_{io}^{(2)}$, w_d is the weight of intermediate measures, u_r^{bad} and u_r^{good} is the weight of y_{ro}^{bad} and y_{ro}^{good}, respectively. The ε also represents a small non-Archimedean number.

Definition 1. *The efficiency of the overall two-stage process is equal to* $E = e_j^1 \cdot e_j^2$.

Definition 2. *The two-stage process is efficient if and only if* $e_j^1 = e_j^2 = 1$.

Here, we assume the two-stage system as a cooperative game model, in which the two stages can be considered as two players in the cooperative game model and in the sequel, we briefly introduce the Nash bargaining game method.

3.2 Nash bargaining game model with shared inputs and undesirable outputs

In this subsection, we will generalize the above results to our model. Therefore, the two players participating in the bargaining are shown by $N = \{1, 2\}$. The payoff vector is defined as an element in two-dimensional Euclidean space R^2. The feasible set $S \subset R^2$ and the breakdown point $b \in R^2$. Then, the bargaining problem can be specified as $\left(N,\ S,\ \overrightarrow{b}\right)$. Hence, we consider the two individual stages as two players in the bargaining manner, the efficiency scores as the payoff and weights selected for efficiency scores as strategies. To launch, one requires to find a breakdown point for stages 1 and 2 which is the starting point for bargaining. It is noted that the breakdown point or status quo displays possible payoff pairs attained if one determines not to bargain with the other player. Thus, the choice of the breakdown point is a matter of modeling judgment (Binmore et al. [3]). We here first make the least ideal DMU and apply its DEA efficiency scores as the breakdown point. In this manner, we consider that if the two stages do not negotiate, their efficiency scores will be defeated. Meanwhile, such a DMU may not exist, however, its inputs and outputs are viewed. We let $\left(x_i^{(1)max}, \alpha x_i^{(2)max}, z_d^{min}\right)$ as the least ideal DMU in the first stage, which consumes the maximum amount of input values, while producing the least amount of output values and intermediate measures. At the same way, we show $\left(\bar{\alpha} x_i^{(2)max}, y_r^{(good)min}, z_d^{min}\right)$ the least ideal DMU in the second stage, which consumes the maximum amount of intermediate measures, additive inputs and undesirable outputs while producing the least amount of output values. We now describe the CRS efficiency for the above two least ideal DMUs is the worst among the present DMUs. The efficiency scores of the two least ideal DMUs for the first and second stage are shown as e_{min}^1 and e_{min}^2, respectively. Also, e_{min}^1 and e_{min}^2 are considered as breakdown point here. Therefore, as it can be seen from Fig. 2 and with respect to relation (3), we present DEA bargaining model for the two-stage network DEA in the presence of the shared inputs and undesirable outputs as follow:

$$
\begin{aligned}
E = \max\ & \left(\frac{\sum_{d=1}^{D} w_d z_{do} + \sum_{r=1}^{R} u_r^{bad} y_{ro}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} - e_{min}^1\right)\left(\frac{\sum_{r=1}^{R} u_r^{good} y_{ro}^{good}}{\sum_{d=1}^{D} w_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} - e_{min}^2\right) \qquad (8)\\
\text{subject to: } & \frac{\sum_{d=1}^{D} w_d z_{do} + \sum_{r=1}^{R} u_r^{bad} y_{ro}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} \geq e_{min}^1,\\
& \frac{\sum_{r=1}^{R} u_r^{good} y_{ro}^{good}}{\sum_{d=1}^{D} w_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)}} \geq e_{min}^2,\\
& \frac{\sum_{d=1}^{D} w_d z_{dj} + \sum_{r=1}^{R} u_r^{bad} y_{rj}^{bad}}{\sum_{i=1}^{I_1} v_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{ij}^{(2)}} \leq 1,\\
& \frac{\sum_{r=1}^{R} u_r^{good} y_{rj}^{good}}{\sum_{d=1}^{D} w_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{ij}^{(2)}} \leq 1,\\
& w_d, u_r^{good}, v_i^{(1)}, v_i^{(2)} \geq \varepsilon, \quad i = 1, \ldots, m; \quad r = 1, \ldots, R; \quad d = 1, \ldots, D,\\
& 0 \leq \alpha, (1-\alpha) = \bar{\alpha} \leq 1,\\
& 0 = L_r \leq u_r^{bad} \leq U_r, \quad r = 1, \ldots, R.
\end{aligned}
$$

In model (8) the feasible set shown by S. Therefore, $(N,\ S,\ \overrightarrow{b})$ for the bargaining problem here can be determined as the triple $\left(\{1,2\}, S, \left\{e_{min}^1,\ e_{min}^2\right\}\right)$, which e_{min}^1 and e_{min}^2 are breakdown point and efficiency scores of stage 1 and stage 2, respectively. First and second constraints are bargaining breakdown point constraints, where the efficiency of each stage must be greater than its breakdown point. Third and fourth constraints are related to efficiency, where the efficiency must be less than one. In the sequel, we will demonstrate that the feasible set S is both compact and convex. In the above formula, $v_i^{(1)}$ is the weight attributed to input $x_{io}^{(1)}$, $v_i^{(2)}$ is the weight attributed to input $x_{io}^{(2)}$, w_d is the weight of intermediate measures, u_r^{bad} and u_r^{good} is the weight of y_{ro}^{bad} and y_{ro}^{good}, respectively. The ε represents also a small non-Archimedean number. Here, the operator of DMU_j can freely assign the shared input resources between the two stages. Then, according to Du et al. [10], the feasible set $S \in R^2$ is both compact and convex. Now, we will prove that the bargaining game model (8) is convex.

Lemma 1. *Feasible set of model (8) is convex.*

Proof. Suppose that

$$\left(v'_1,\ldots,v'_{I_1},v'_1,\ldots,v'_{I_2},u'_{(bad)1},\ldots,u'_{(bad)R},,u'_{(good)1},\ldots,u'_{(good)R},\omega'_1,\ldots,\omega'_d\right)\in S,$$
$$\left(v''_1,\ldots,v''_{I_1},v''_1,\ldots,v''_{I_2},u''_{(bad)1},\ldots,u''_{(bad)R},u''_{(good)1},\ldots,u''_{(good)R},\omega''_1,\ldots,\omega''_d\right)\in S.$$

For any $\lambda \in [0,1]$ we have:

$$\begin{aligned}
&1)\,\lambda v'_{i_1}+(1-\lambda)v''_{i_1}\geq 0\,, && i_1=1,\ldots,I_1,\\
&2)\,\lambda v'_{i_2}+(1-\lambda)v''_{i_2}\geq 0, && i_2=1,\ldots,I_2,\\
&3)\,\lambda u'_{r(bad)}+(1-\lambda)u''_{r(bad)}\geq 0\,, && r=1,\ldots,R,\\
&4)\,\lambda u'_{r(good)}+(1-\lambda)u''_{r(good)}\geq 0\,, && r=1,\ldots,R,\\
&5)\,\lambda \omega'_d+(1-\lambda)\omega''_d\geq 0, && d=1,\ldots,D.
\end{aligned}$$

Since $\sum_{i=1}^{I_1} v_{ij}^{(1)}x_i^{(1)}>0, \sum_{i=1}^{I_2} v_{ij}^{(2)}x_i^{(2)}>0, \sum_{d=1}^{D}\omega_d z_d>0, \sum_{r=1}^{R}u_{r(bad)}y_{r(bad)}>0$ and $\sum_{r=1}^{R}u_{r(good)}y_{r(good)}>0$ for all $DMU_j\,(j=1,\ldots,n)$ we define the following constraints in $S\in R^2$.

1) $\frac{\sum_{d=1}^{D}w_d z_{do}+\sum_{r=1}^{R}u_r^{bad}y_{ro}^{bad}}{\sum_{i=1}^{I_1}v_i^{(1)}x_{io}^{(1)}+\alpha\sum_{i=1}^{I_2}v_i^{(2)}x_{io}^{(2)}}\geq e_{min}^1 \Rightarrow \sum_{d=1}^{D}w_d z_{do}+\sum_{r=1}^{R}u_r^{bad}y_{ro}^{bad}\geq e_{min}^1\sum_{i=1}^{I_1}v_i^{(1)}x_{io}^{(1)}+\alpha\sum_{i=1}^{I_2}v_i^{(2)}x_{io}^{(2)}$,

2) $\frac{\sum_{r=1}^{R}u_r^{good}y_{ro}^{good}}{\sum_{d=1}^{D}w_d z_{do}+\bar{\alpha}\sum_{i=1}^{I_2}v_i^{(2)}x_{io}^{(2)}}\geq e_{min}^2 \Rightarrow \sum_{r=1}^{R}u_r^{good}y_{ro}^{good}\geq e_{min}^2\sum_{d=1}^{D}w_d z_{do}+\bar{\alpha}\sum_{i=1}^{I_2}v_i^{(2)}x_{io}^{(2)}$,

3) $\frac{\sum_{d=1}^{D}w_d z_{dj}+\sum_{r=1}^{R}u_r^{bad}y_{rj}^{bad}}{\sum_{i=1}^{I_1}v_i^{(1)}x_{ij}^{(1)}+\alpha\sum_{i=1}^{I_2}v_i^{(2)}x_{ij}^{(2)}}\leq 1 \Rightarrow \sum_{d=1}^{D}w_d z_{dj}+\sum_{r=1}^{R}u_r^{bad}y_{rj}^{bad}\leq \sum_{i=1}^{I_1}v_i^{(1)}x_{ij}^{(1)}+\alpha\sum_{i=1}^{I_2}v_i^{(2)}x_{ij}^{(2)}$,

4) $\frac{\sum_{r=1}^{R}u_r^{good}y_{rj}^{good}}{\sum_{d=1}^{D}w_d z_{dj}+\bar{\alpha}\sum_{i=1}^{I_2}v_i^{(2)}x_{ij}^{(2)}}\leq 1, \Rightarrow \sum_{r=1}^{R}u_r^{good}y_{rj}^{good}\leq \sum_{d=1}^{D}w_d z_{dj}+\bar{\alpha}\sum_{i=1}^{I_2}v_i^{(2)}x_{ij}^{(2)}$.

Then, we have

$$\begin{aligned}
&\sum_{d=1}^{D}\left[\lambda w'_d+(1-\lambda)w''_d\right]z_{dj}+\sum_{r=1}^{R}\left[\lambda u'_{r(bad)}+(1-\lambda)u''_{r(bad)}\right]y_{rj}^{bad}\\
&\quad=\lambda\sum_{d=1}^{D}w'_d z_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_d z_{dj}+\lambda\sum_{r=1}^{R}u'_{r(bad)}y_{rj}^{bad}+(1-\lambda)\sum_{r=1}^{R}u''_{r(bad)}y_{rj}^{bad}\\
&\quad\leq\lambda\sum_{i=1}^{I_1}v'^{(1)}_i x_{ij}^{(1)}+(1-\lambda)\sum_{i=1}^{I_1}v''^{(1)}_i x_{ij}^{(1)}+\lambda\alpha\sum_{i=1}^{I_2}v'^{(2)}_i x_{ij}^{(2)}+(1-\lambda)\alpha\sum_{i=1}^{I_2}v''^{(2)}_i x_{ij}^{(2)}\\
&\quad=\sum_{i=1}^{I_1}\left[\lambda v'^{(1)}_i+(1-\lambda)v''^{(1)}_i\right]x_{ij}^{(1)}+\alpha\sum_{i=1}^{I_2}\left[\lambda v'^{(2)}_i+(1-\lambda)v''^{(2)}_i\right]x_{ij}^{(2)},
\end{aligned}$$

and

$$\begin{aligned}\sum_{r=1}^{R}\left[\lambda\,u'_{r(good)}+(1-\lambda)\,u''_{r(good)}\right]y_{rj}^{good}&=\lambda\sum_{r=1}^{R}u'_{r(good)}\,y_{rj}^{good}+(1-\lambda)\sum_{r=1}^{R}u''_{r(good)}\,y_{rj}^{good}\\&\leq\lambda\sum_{d=1}^{D}w'_{d}\,z_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_{d}\,z_{dj}+\lambda\bar{\alpha}\sum_{i=1}^{I_2}v'^{(2)}_{i}\,x^{(2)}_{ij}+(1-\lambda)\bar{\alpha}\sum_{i=1}^{I_2}v''^{(2)}_{i}\,x^{(2)}_{ij}\\&=\sum_{d=1}^{D}\left[\lambda\,w'_{d}+(1-\lambda)\,w''_{d}\right]z_{dj}+\bar{\alpha}\sum_{i=1}^{I_2}\left[\lambda\,v'^{(2)}_{i}+(1-\lambda)\,v''^{(2)}_{i}\right]x^{(2)}_{ij}.\end{aligned}$$

Similarly, we have

$$\begin{aligned}&\sum_{d=1}^{D}\left[\lambda\,w'_{d}+(1-\lambda)\,w''_{d}\right]z_{dj}+\sum_{r=1}^{R}\left[\lambda\,u'_{r(bad)}+(1-\lambda)\,u''_{r(bad)}\right]y_{rj}^{bad}\\&\quad=\lambda\sum_{d=1}^{D}w'_{d}\,z_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_{d}\,z_{dj}+\lambda\sum_{r=1}^{R}u'_{r(bad)}\,y_{rj}^{bad}+(1-\lambda)\sum_{r=1}^{R}u''_{r(bad)}\,y_{rj}^{bad}\\&\quad\geq e^{1}_{min}(\lambda\sum_{i=1}^{I_1}v'^{(1)}_{i}\,x^{(1)}_{ij}+(1-\lambda)\sum_{i=1}^{I_1}v''^{(1)}_{i}\,x^{(1)}_{ij}+\lambda\alpha\sum_{i=1}^{I_2}v'^{(2)}_{i}\,x^{(2)}_{ij}+(1-\lambda)\alpha\sum_{i=1}^{I_2}v''^{(2)}_{i}\,x^{(2)}_{ij})\\&\quad=\sum_{i=1}^{I_1}\left[\lambda\,v'^{(1)}_{i}+(1-\lambda)\,v''^{(1)}_{i}\right]x^{(1)}_{ij}+\alpha\sum_{i=1}^{I_2}\left[\lambda\,v'^{(2)}_{i}+(1-\lambda)\,v''^{(2)}_{i}\right]x^{(2)}_{ij},\end{aligned}$$

and

$$\begin{aligned}\sum_{r=1}^{R}\left[\lambda\,u'_{r(good)}+(1-\lambda)\,u''_{r(good)}\right]y_{rj}^{good}&=\lambda\sum_{r=1}^{R}u'_{r(good)}\,y_{rj}^{good}+(1-\lambda)\sum_{r=1}^{R}u''_{r(good)}\,y_{rj}^{good}\\&\geq e^{1}_{min}(\lambda\sum_{d=1}^{D}w'_{d}\,z_{dj}+(1-\lambda)\sum_{d=1}^{D}w''_{d}\,z_{dj}+\lambda\bar{\alpha}\sum_{i=1}^{I_2}v'^{(2)}_{i}\,x^{(2)}_{ij}+(1-\lambda)\bar{\alpha}\sum_{i=1}^{I_2}v''^{(2)}_{i}\,x^{(2)}_{ij})\\&=\sum_{d=1}^{D}\left[\lambda\,w'_{d}+(1-\lambda)\,w''_{d}\right]z_{dj}+\bar{\alpha}\sum_{i=1}^{I_2}\left[\lambda\,v'^{(2)}_{i}+(1-\lambda)\,v''^{(2)}_{i}\right]x^{(2)}_{ij}.\end{aligned}$$

Therefore, we have

$$\begin{pmatrix}\lambda\,v'_{i\in I_1}+(1-\lambda)\,v''_{i\in I_1},\lambda\,v'_{i\in I_2}+(1-\lambda)\,v''_{i\in I_2},\lambda\,\omega'_{d}+(1-\lambda)\,\omega''_{d},\\ \lambda u'_{r(bad)}+(1-\lambda)u''_{r(bad)}\,\lambda u'_{r(good)}+(1-\lambda)u''_{r(good)}\end{pmatrix}\in S,$$

where $i=1,\ldots,I_1, r=1,\ldots,R$ and $d=1,\ldots,D$, or equivalently,

$$\begin{aligned}&\lambda\left(v'_{1},\ldots,v'_{I_1},v'_{1},\ldots,v'_{I_2},u'_{1(bad)},\ldots,u'_{R(bad)},u'_{1(good)},\ldots,u'_{R(good)},\omega'_{1},\ldots,\omega'_{d}\right)\\&+(1-\lambda)\left(v''_{1},\ldots,v''_{I_1},v''_{1},\ldots,v''_{I_2},u''_{1(bad)},\ldots,u''_{(bad)R},u''_{1(good)},\ldots,u''_{(good)R},\omega''_{1},\ldots,\omega''_{d}\right)\in S.\end{aligned}$$

Consequently, S is a convex set. □

Now, consider linearizing model (8). Let

$$\sum_{i=1}^{I_1} v_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)} = \frac{1}{t_1} \quad \Rightarrow \quad t_1 = \left(\sum_{i=1}^{I_1} v_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)} \right)^{-1}, \tag{9}$$

and

$$\sum_{d=1}^{D} w_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)} = \frac{1}{t_2} \quad \Rightarrow \quad t_2 = \left(\sum_{d=1}^{D} w_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} v_i^{(2)} x_{io}^{(2)} \right)^{-1},$$

where

$$t_1 \cdot w_d = \omega_d, \quad t_1 \cdot u_r^{bad} = \mu_r^{bad}, \quad t_1 \cdot v_i^{(1)} = \upsilon_i^{(1)}, \quad t_1 \cdot v_i^{(2)} = \upsilon_i^{(2)}, \quad t_2 \cdot u_r^{good} = \mu_r^{good}. \tag{10}$$

Then model (8) can be converted into the following model:

$$\begin{aligned} E = \ & \max \left(\sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} - e_{min}^1 \right) \left(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} - e_{min}^2 \right) \\ \text{subject to :} \ & \sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\ & \sum_{d=1}^{D} \omega_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\ & \sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \geq e_{min}^1, \\ & \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \geq e_{min}^2, \\ & \sum_{d=1}^{D} \omega_d z_{dj} + \sum_{r=1}^{R} \mu_r^{bad} y_{rj}^{bad} - \left(\sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\ & \sum_{r=1}^{R} \mu_r^{good} y_{rj}^{good} - \left(\sum_{d=1}^{D} \omega_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\ & \omega_d, \mu_r^{good}, \upsilon_i^{(1)}, \upsilon_i^{(2)} \geq \varepsilon, \quad i = 1, \ldots, m, \quad r = 1, \ldots, R, \quad d = 1, \ldots, D, \\ & 0 \leq \alpha, (1-\alpha) = \bar{\alpha} \leq 1, \\ & 0 = L_r \leq \mu_r^{bad} \leq U_r, \quad r = 1, \ldots, R, \end{aligned} \tag{11}$$

or

$$\begin{aligned} E = \max & \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \cdot \left(\sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \right) - e_{min}^1 \cdot \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \\ & - e_{min}^2 \cdot \left(\sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \right) + e_{min}^1 \cdot e_{min}^2, \end{aligned} \tag{12}$$

$$\begin{aligned}
\text{subject to}: & \sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\
& \sum_{d=1}^{D} \omega_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\
& \sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \geq e_{min}^1, \\
& \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \geq e_{min}^2, \\
& \sum_{d=1}^{D} \omega_d z_{dj} + \sum_{r=1}^{R} \mu_r^{bad} y_{rj}^{bad} - \left(\sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\
& \sum_{r=1}^{R} \mu_r^{good} y_{rj}^{good} - \left(\sum_{d=1}^{D} \omega_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\
& \omega_d, \mu_r^{good}, \upsilon_i^{(1)}, \upsilon_i^{(2)} \geq \varepsilon, \quad i = 1, \ldots, m, \quad r = 1, \ldots, R, \quad d = 1, \ldots, D, \\
& 0 \leq \alpha, (1-\alpha) = \bar{\alpha} \leq 1, \\
& 0 = L_r \leq \mu_r^{bad} \leq U_r, \quad r = 1, \ldots, R.
\end{aligned}$$

We further have

$$\begin{aligned}
E = \max\ & \Big(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \times \sum_{d=1}^{D} \omega_d z_{do}\Big) + \Big(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad}\Big) \\
& - e_{min}^1 \cdot \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} - e_{min}^2 \times \sum_{d=1}^{D} \omega_d z_{do} - e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2 \qquad (13) \\
\text{subject to}: & \sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\
& \sum_{d=1}^{D} \omega_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1, \\
& \sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \geq e_{min}^1, \\
& \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \geq e_{min}^2, \\
& \sum_{d=1}^{D} \omega_d z_{dj} + \sum_{r=1}^{R} \mu_r^{bad} y_{rj}^{bad} - \left(\sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\
& \sum_{r=1}^{R} \mu_r^{good} y_{rj}^{good} - \left(\sum_{d=1}^{D} \omega_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0, \\
& \omega_d, \mu_r^{good}, \upsilon_i^{(1)}, \upsilon_i^{(2)} \geq \varepsilon, \quad d = 1, \ldots, D, \quad r = 1, \ldots, R, \quad i = 1, \ldots, m, \\
& 0 \leq \alpha, (1-\alpha) = \bar{\alpha} \leq 1,
\end{aligned}$$

$$0 = L_r \leq \mu_r^{bad} \leq U_r, \quad r = 1, \ldots, R.$$

The objective function

$$E = \max\left\{(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad}) - e_{min}^1 \cdot \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good}\right\} + \left\{(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \times \sum_{d=1}^{D} \omega_d z_{do}) - e_{min}^2 \times \sum_{d=1}^{D} \omega_d z_{do} - e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2\right\},$$

can be written as

$$E = \max\left\{\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} (\sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} - e_{min}^1)\right\} + \left\{\sum_{d=1}^{D} \omega_d z_{do} (\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} - e_{min}^2)\right\} + \left\{-e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2\right\},$$

since

$$\sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \geq e_{min}^1.$$

We have

$$\sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} - e_{min}^1 = \sum_{d=1}^{D} \omega_d z_{do},$$

and therefore

$$E = \max\left\{(\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \times \sum_{d=1}^{D} \omega_d z_{do}) + (\sum_{d=1}^{D} \omega_d z_{do} \times \sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good}) + (-e_{min}^2 \times \sum_{d=1}^{D} \omega_d z_{do})\right\} + \left\{-e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2\right\}.$$

Also $\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \geq e_{min}^2$. Then

$$E = \max\left\{e_{min}^2 \times \sum_{d=1}^{D} \omega_d z_{do} - e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2\right\}.$$

Hence, we obtain model (14) as follows:

$$E = \max\left\{e_{min}^2 \times \sum_{d=1}^{D} \omega_d z_{do} - e_{min}^2 \times \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} + e_{min}^1 \cdot e_{min}^2\right\} \tag{14}$$

$$\text{subject to}: \sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{io}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1,$$

$$
\begin{aligned}
&\sum_{d=1}^{D} \omega_d z_{do} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{io}^{(2)} = 1,\\
&\sum_{d=1}^{D} \omega_d z_{do} + \sum_{r=1}^{R} \mu_r^{bad} y_{ro}^{bad} \geq e_{min}^1,\\
&\sum_{r=1}^{R} \mu_r^{good} y_{ro}^{good} \geq e_{min}^2,\\
&\sum_{d=1}^{D} \omega_d z_{dj} + \sum_{r=1}^{R} \mu_r^{bad} y_{rj}^{bad} - \left(\sum_{i=1}^{I_1} \upsilon_i^{(1)} x_{ij}^{(1)} + \alpha \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0,\\
&\sum_{r=1}^{R} \mu_r^{good} y_{rj}^{good} - \left(\sum_{d=1}^{D} \omega_d z_{dj} + \bar{\alpha} \sum_{i=1}^{I_2} \upsilon_i^{(2)} x_{ij}^{(2)} \right) \leq 0,\\
&\omega_d, \mu_r^{good}, \upsilon_i^{(1)}, \upsilon_i^{(2)} \geq \varepsilon, \quad i = 1,\dots,m, \quad r = 1,\dots,R, \quad d = 1,\dots,D,\\
&0 \leq \alpha, (1-\alpha) = \bar{\alpha} \leq 1,\\
&0 = L_r \leq \mu_r^{bad} \leq U_r, \quad r = 1,\dots,R.
\end{aligned}
$$

In the next section, a numerical example is given to evaluate the proposed model.

4 An illustrative application

Suppose there is a two-stage production process in which there are two types of inputs. We denote, for each DMU_j $(j = 1,\dots,n)$, $x_{ij}^{(1)}$ $(i = 1,\dots,I_1)$ and $\alpha x_{ij}^{(2)}$ $(i = 1,\dots,I_2)$ as two inputs to stage 1. The output of stage 1 is denoted by y_{rj}^{bad} $(i = r,\dots,R)$ and z_{dj} $(d = 1,\dots,D)$. The output of stage 2 is denoted by y_{rj}^{good} $(r = 1,\dots,R)$ and the intermediate measures is denoted by z_{dj} $(d = 1,\dots,D)$. In stage 2, we denote z_{dj} $(d = 1,\dots,D)$, $\overline{\alpha} x_{ij}^{(2)}$ $(i = 1,\dots,I_2)$ and y_{rj}^{good} $(r = 1,\dots,R)$ as inputs and outputs for stage 2, respectively. In the Table 3, each DMU_j allocates some parts α of shared input sources to the first stage and the remaining $(1-\alpha) = \overline{\alpha}$ to the second stage.

Table 1 and Table 4 show the data set. The results from the proposed model (14) is reported in Tables 2, 3, 5. The E^* is the value of efficiency score of the whole operation of the system, e_{min}^1 and e_{min}^2 are efficiency scores of stage 1 and stage 2. Table 2, when $\theta_1 = 1$, illustrates the change of E^* with $e_{min}^1 = 1$, so that the efficiency values of DMUs with change of e_{min}^1 can be changed within $(0,1]$. Here, based on change of e_{min}^1, in each step E^* can be changed. When θ_{min}^1=1 , DMU05 has $E^* = 1$.

Table 3 illustrates the change of E^* with $e_{min}^2 = 1$, so that the efficiency values of DMUs with change of e_{min}^2 can be changed within $(0,1]$. Here, based on change of e_{min}^2, in each step E^* can be changed. When θ_{min}^2=1, DMU02 has $E^* = 1$.

5 Conclusions

In this paper, we proposed a bargaining game-DEA model for assessing relative efficiency score of network structure processes. We addressed the issue of conflicts between stages, with shared inputs and undesirable outputs in the first stage. For each stage, a bargaining game structure was considered. Each

Table 1: The data set.

DMU	$x_{ij}^{(1)}$	$x_{ij}^{(2)}$	Z_{dj}	y_{rj}^{bad}	y_{rj}^{good}
DMU01	854	214	368	324	321
DMU02	125	369	400	245	254
DMU03	654	842	861	632	216
DMU04	587	654	258	214	362
DMU05	326	265	251	421	392

Table 2: The results for the data set of Table 1, when $\theta_1 = 1$.

DMU	$e_{min}^{(1)}$	$e_{min}^{(2)}$	E	α	$\overline{\alpha}$
DMU01	1	0.9642	0.9821	0.3658	0.6342
DMU02	1	0.4265	0.7658	0.5523	0.4477
DMU03	1	0.2458	0.6324	0.1479	0.8521
DMU04	1	0.6987	0.7888	0.5000	0.5000
DMU05	1	1.0000	1.0000	0.5013	0.4987

Table 3: The results for the data set of Table 1, when $\theta_2 = 1$.

DMU	$e_{min}^{(1)}$	$e_{min}^{(2)}$	E	α	$\overline{\alpha}$
DMU01	0.7542	1	0.8574	0.8541	0.1459
DMU02	1.0000	1	1.0000	0.2555	0.7445
DMU03	0.3654	1	0.6358	0.6478	0.3522
DMU04	0.8354	1	0.9687	0.3693	0.6307
DMU05	0.8745	1	0.8888	0.7821	0.2179

Table 4: The data set.

DMU	$x_{ij}^{(1)}$	$x_{ij}^{(2)}$	Z_{dj}	y_{rj}^{bad}	y_{rj}^{good}
DMU01	3698	5369	752	656	3564
DMU02	5246	5412	326	5356	2145
DMU03	7145	2148	542	354	3698
DMU04	3369	4532	632	542	5246
DMU05	5214	4521	623	652	7145
DMU06	5369	3678	354	752	3369
DMU07	5412	4213	654	326	5214
DMU08	2148	2846	423	542	5369

stage is assumed to be a player in Nash bargaining game. Hence, the proposed model can determine which DMU in which stage has better performance. Also, in the offered model, it can be possible to

Table 5: The results for the data set of Table 4, when $\alpha = 0.5$.

DMU	$e_{min}^{(1)}$	$e_{min}^{(2)}$	E
DMU01	0.6547	0.8654	0.7452
DMU02	0.9856	0.8456	0.9012
DMU03	0.7568	0.9654	0.8532
DMU04	0.6358	0.9214	0.7832
DMU05	1.0000	1.0000	1.0000
DMU06	0.9632	0.4567	0.6398
DMU07	0.9214	0.8524	0.8832
DMU08	0.7569	0.7564	0.7564

gain a set of convenient portion for sharing the inputs between the stages in the presence of undesirable outputs and to determine whether intermediate products should be processed further at the split-off point or not. The results of the case study shows more practical and managerial views to managers and policy makers.

The current models are under the assumption of CRS, how to modify these models for general network structure by variable return to scale assumption is also a direction for future research. Another interesting direction of research is modeling the proposed structure with a perspective of dynamic effects and investigating the relative efficiency of each stage. Finally, in future empirical analysis on this subject, the proposed framework can also be applied to other complex production processes or service processes.

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